Localization of two retailers in a single urban area

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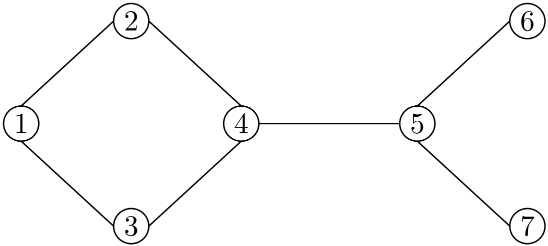
- A total of N households are located in the urban area
- Each household represents a consumer and shops at the retailer with the minimum total purchase costs. If two retailers offer the same total purchase costs, it will shop at both retailers alternately
- Each household can become a retailer. When becoming a retailer, the household remains a consumer as well. Only one retailer can be located at each household
- Distances between neighboring households are equidistant and equal to 1
- Two retailers A and B will locate their retail units in the urban area with prices p and Δp respectively

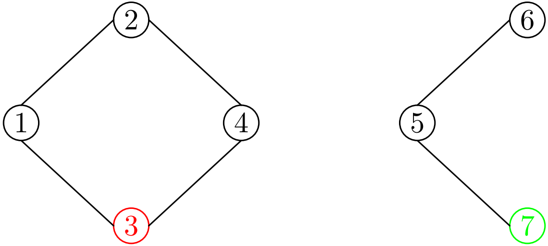
Graph of the urban area



$G(V, H)$

- $G = (V, H)$ - map of the urban area
- V - set of vertices representing households, $|V| = N$
- H - set of edges representing neighbouring households

Retailer subgraphs



Retailer A

$G_A(V^A, H^A)$

Retailer B

$G_B(V^B, H^B)$

- $G_A(V^A, H^A) \subseteq G(V, H)$ - subset induced by vertex subset $V^A \subseteq V$, consumers shopping at retailer A
- $G_B(V^B, H^B) \subseteq G(V, H)$ - subset induced by vertex subset $V^B \subseteq V$, consumers shopping at retailer B

Sequential game form

- $\{\{A, B\}; \chi^A, \chi^B; \Pi^A, \Pi^B\}$ - sequential two player game
- $\{A, B\}$ - players representing the two retailers
- $\chi^A = \{1, 2, \ldots, N\}$ - strategy space of player A
- $\chi^B = \{1, 2, \ldots, N\} \setminus a$ - strategy space of player B
- a, b - strategies selected by player A and player B respectively
- Π^A, Π^B - payoff functions of players

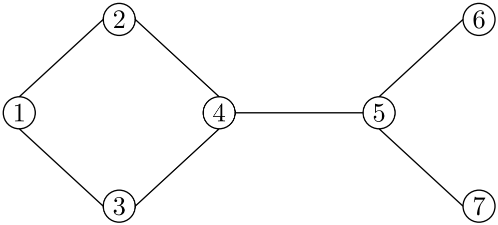
Payoff functions

- d - distance function between two vertices
- $V^A(G_A) = V^A_{(a,b)} = \{i \in V | d(i;a) \leq d(i;b) + \Delta p\}$
- $V^B(G_B) = V^B_{(a,b)} = \{i \in V | d(i;b) + \Delta p \leq d(i;a)\}$
- $\Pi^A(G_A) = \Pi^A_{(a,b)} = p\left(|V^A_{(a,b)}| + \frac{|V^A_{(a,b)}|+|V^B_{(a,b)}|-|V|}{2}\right)$
  - payoff function of player A
- $\Pi^B(G_B) = \Pi^B_{(a,b)} = (p+\Delta p)\left(|V^B_{(a,b)}| + \frac{|V^A_{(a,b)}|+|V^B_{(a,b)}|-|V|}{2}\right)$
  - payoff function of player B

Equilibrium localization

- to find the subgame perfect Nash equilibrium strategy pair (a^*, b^*) we have to first find the equilibrium strategy b_a^* of player B for every possible strategy $\forall a \in \chi^A$ of player A:
$(\forall b \in \chi^B)\left(\Pi^B_{(a,b_a^*)} \geq \Pi^B_{(a,b)}\right)$
- using backwards induction player A selects the strategy a^*:
$(\forall a \in \chi^A)\left(\Pi^A_{(a^*,b_a^*)} \geq \Pi^A_{(a,b_a^*)}\right)$
- Since player A opens his store without knowing the price of player B at that time, he will presume that player B will enter the market with $\Delta p = 0$, as this represents the worst case scenario for player A

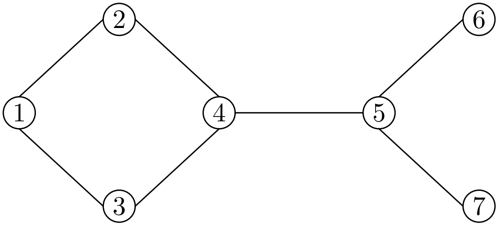
Payoff matrices for $p = 1$ and $\Delta p = 0$



Payoff matrix Π^A

$a \backslash b$	1	2	3	4	5	6	7
1	-	2	2	2	3	3.5	3.5
2	5	-	3.5	2	3	4	4
3	5	3.5	-	2	3	4	4
4	5	5	5	-	4	5	5
5	4	4	4	3	-	6	6
6	3.5	3	3	2	1	-	3.5
7	3.5	3	3	2	1	3.5	-

Payoff matrices for $p = 1$ and $\Delta p = 0$



Payoff matrix Π^B

$a \backslash b$	1	2	3	4	5	6	7
1	-	5	5	5	4	3.5	3.5
2	2	-	3.5	5	4	3	3
3	2	3.5	-	5	4	3	3
4	2	2	2	-	3	2	2
5	3	3	3	4	-	1	1
6	3.5	4	4	5	6	-	3.5
7	3.5	4	4	5	6	3.5	-

Finding the equilibrium strategy pair

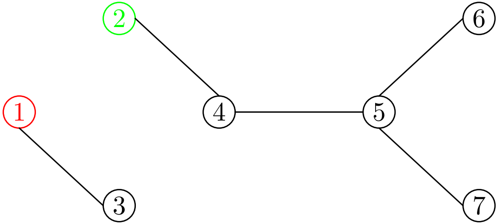
Payoff matrix Π^A

$a \backslash b$	1	2	3	4	5	6	7	$\min_{b\in\chi^B} \Pi^A_{(a,b)}$
1	-	2	2	2	3	3.5	3.5	2
2	5	-	3.5	2	3	4	4	2
3	5	3.5	-	2	3	4	4	2
4	5	5	5	-	4	5	5	4
5	4	4	4	3	-	6	6	3
6	3.5	3	3	2	1	-	3.5	1
7	3.5	3	3	2	1	3.5	-	1

$\Pi^A_{(a^*,b^*)} = \max_{a\in\chi^A}\left(\min_{b\in\chi^B} \Pi^A_{(a,b)}\right)$ and

$\min_{b\in\chi^B} \Pi^A_{(a^*,b)} = \max_{a\in\chi^A}\left(\min_{b\in\chi^B} \Pi^A_{(a,b)}\right)$

Possible reactions of retailer B



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$a \backslash b$	1	2	3	4	5	6	7	$\min_{b \in \chi^B} \Pi^A_{(a,b)}$
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3	5	3.5	-	2	3	4	4	2
4	5	5	5	-	4	5	5	4
5	4	4	4	3	-	6	6	3
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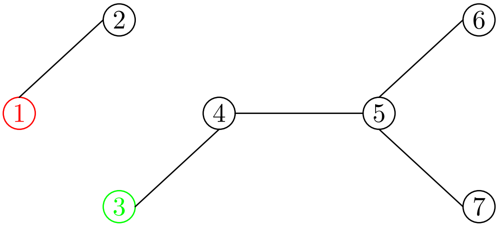

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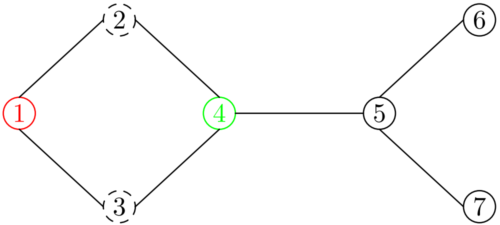

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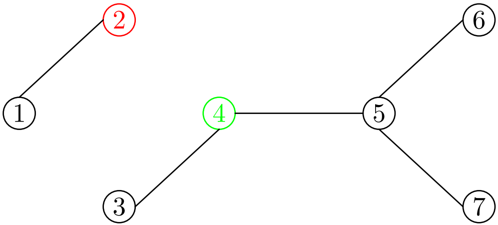

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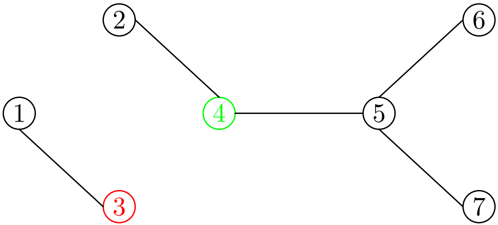

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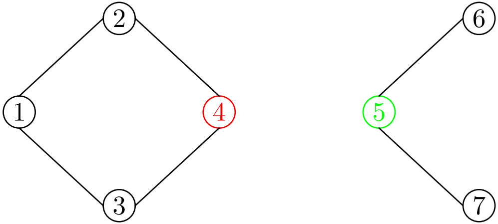

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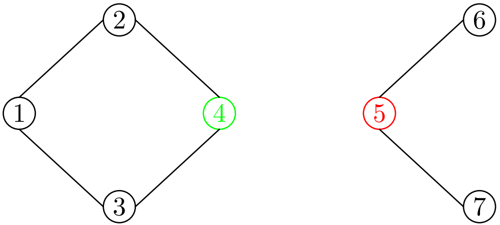

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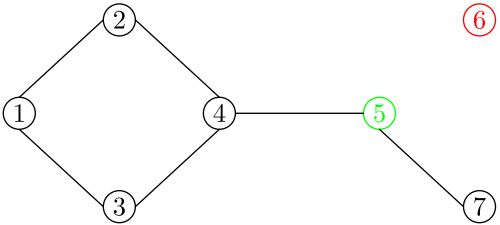

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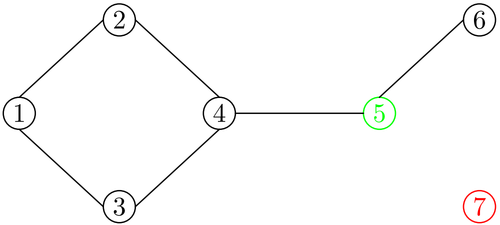

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Finding the equilibrium strategy pair

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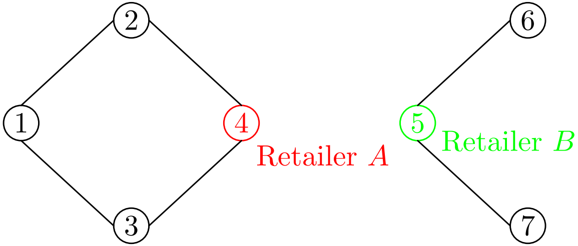
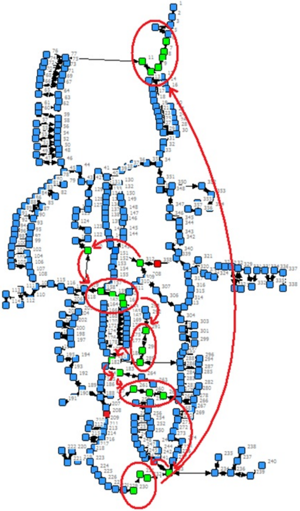
Finding the equilibrium strategy pair



Equilibrium strategy pair

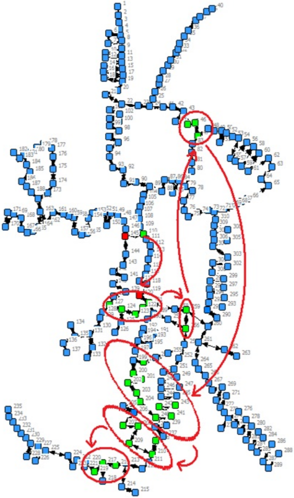
$(a^*, b^*) = (4; 5), \Pi^A_{(4,5)} = 4, \Pi^B_{(4,5)} = 3$

Haniska



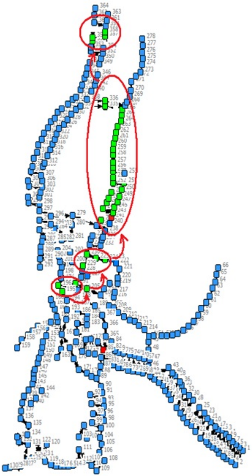
Commodity	Retailer 1	Retailer 2
Bread	1,3	1,43
Milk	0,59	0,68
Beer	0,58	0,65
Pastry	0,06	0,07
Bun	0,25	0,37
Rice	0,89	0,95
Butter	0,55	0,76
Mineral water	0,49	0,48
Salt	0,25	0,25
Sugar	1,25	1,16
Flour	0,42	0,57
Toilet paper	0,21	0,22
Floor space	150	50

Kavečany



Commodity	Retailer 1	Retailer 2
Bread	1,1	1,15
Milk	0,64	0,65
Beer	0,66	0,66
Pastry	0,06	0,09
Bun	0,37	0,49
Rice	0,84	0,98
Butter	0,71	0,69
Mineral water	0,46	0,55
Salt	0,28	0,29
Sugar	1,38	1,33
Flour	0,5	0,59
Toilet paper	0,2	0,28
Floor space	70	50

Zemplínske Hradište



Commodity	Retailer 1	Retailer 2
Bread	1	1,1
Milk	0,59	0,78
Beer	0,63	0,65
Pastry	0,05	0,09
Bun	0,32	0,4
Rice	0,95	1,1
Butter	0,89	0,95
Mineral water	0,55	0,69
Salt	0,27	0,35
Sugar	1,42	1,3
Flour	0,48	0,55
Toilet paper	0,26	0,27
Floor space	85	24

Thank you for your attention

Contact

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