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# Security protocols

Lecture 7

Key Agreement

# Diffie-Hellman Key Agreement

- Shared prime  $p$  and generator  $g$  of  $Z_p^*$  ;  $ra, rb$  – private random
  - 1. A  $\rightarrow$  B:  $g^{ra} \bmod p$
  - 2. B  $\rightarrow$  A:  $g^{rb} \bmod p$
$$K = (g^{ra})^{rb} \bmod p = (g^{rb})^{ra} \bmod p = g^{ra \cdot rb} \bmod p$$
- intractability of Diffie-Hellman problem (DDH)

# Diffie-Hellman Key Agreement

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$$K = (g^{ra})^{rb} \bmod p = (g^{rb})^{ra} \bmod p = g^{ra*rb} \bmod p$$
- intractability of Diffie-Hellman problem (DDH)
- man-in-the-middle attack
  - 1. A  $\rightarrow C_B$  :  $g^{ra} \bmod p$
  - 1.'  $C_A \rightarrow B$  :  $g^{rc} \bmod p$
  - 2.' B  $\rightarrow C_A$  :  $g^{rb} \bmod p$
  - 2.  $C_B \rightarrow A$  :  $g^{rc} \bmod p$
$$K_{AC} = g^{ra*rc} \bmod p ; K_{CB} = g^{rc*rb} \bmod p$$

# One-Pass Key Establishment with ElGamal encr.

- $x_a, x_b$  – private DH long-term keys
- $y_a = g^{x_a} \bmod p$  ;  $y_b = g^{x_b} \bmod p$ 
  - A  $\rightarrow$  B :  $g^{r_a} \bmod p$

$$K = g^{r_a \cdot x_b} \bmod p = y_b^{r_a} \bmod p$$

freshness of  $g^{r_a} \bmod p$  ?

# One-Pass Key Establishment with ElGamal encr.

- $x_a, x_b$  – private DH long-term keys
- $y_a = g^{x_a} \bmod p$  ;  $y_b = g^{x_b} \bmod p$

(Agnew, Mullin, Vanstone)

- A  $\rightarrow$  B :  $g^{ra} \bmod p$ ,  $s = t * y_b^{ra} \bmod p$   
 $t = s * \text{inv}(g^{ra * x_b}) \bmod p$  ;  $K = s^t \bmod p = (g^{x_a * x_b})^t \bmod p$

(Nyberg, Rueppel) t - time

- A  $\rightarrow$  B :  $g^{ra} \bmod p$ ,  $s = ra - x_a * \#(t, y_b^{ra} \bmod p) \bmod p$   
 $K = g^{ra * x_b} \bmod p$  ; verify  $g^{ra} \bmod p \stackrel{?}{=} g^s y_a^{\#(t, K)}$

(Lim, Lee)

1. A  $\rightarrow$  B:  $\{Na\}S$
2. B  $\rightarrow$  A:  $\{A, Nb\}(S \text{ xor } Na)$
3. A  $\rightarrow$  B:  $\{Na\}(S \text{ xor } Na)$        $S = f(g^{x_a * x_b} \bmod p)$  ;  $K = Na \text{ xor } Nb$

# Diffie-Hellman Authenticated Key Exchange

- $x_a, x_b$  – private long-term keys
- $y_a = g^{x_a} \bmod p$  ;  $y_b = g^{x_b} \bmod p$

MTI (Matsumoto-Takashima-Imai) (1986)

- A  $\rightarrow$  B:  $g^{r_a} \bmod p$
- B  $\rightarrow$  A:  $g^{r_b} \bmod p$

$$K = (g^{r_b})^{x_a} y_b^{r_a} \bmod p = (g^{r_a})^{x_b} y_a^{r_b} \bmod p = (g^{r_a x_b + r_b x_a}) \bmod p$$

KEA (Key Exchange Algorithm) – NSA (1998)

- A  $\rightarrow$  B:  $g^{r_a} \bmod p$
- B  $\rightarrow$  A:  $g^{r_b} \bmod p$

$$K = (g^{r_b})^{x_a} + y_b^{r_a} \bmod p = (g^{r_a})^{x_b} + y_a^{r_b} \bmod p$$

+ check  $g^{r_b}, y_b$  in subgroup  $G$  of order  $q$   $q \mid (p-1)$

# Diffie-Hellman Authenticated Key Exchange

$x_a, x_b$  – private long-term keys;  $y_a = g^{x_a} \bmod p$ ;  $y_b = g^{x_b} \bmod p$

## Unified Model Protocol (NIST SP-800)

- A  $\rightarrow$  B:  $g^{r_a} \bmod p$
  - B  $\rightarrow$  A:  $g^{r_b} \bmod p$
- $$K = ((g^{r_b})^{r_a} \bmod p \parallel g^{x_a * x_b} \bmod p) = ((g^{r_a})^{r_b} \bmod p \parallel g^{x_a * x_b} \bmod p)$$

## MQV protocol (IEEE P1363-2000)

- A  $\rightarrow$  B:  $g^{r_a} \bmod p$        $t_a = (g^{r_a} \bmod p)_w$       low  $w$  bits
  - B  $\rightarrow$  A:  $g^{r_b} \bmod p$        $t_b = (g^{r_b} \bmod p)_w$
- $$s_a = (r_a + t_a * x_a) \bmod q; \quad s_b = (r_b + t_b * x_b) \bmod q \quad q \mid (p-1)$$
- $$K = (g^{r_b} y_b^{t_b})^{s_a} \bmod p = (g^{r_b} y_a^{t_a})^{s_b} \bmod p$$
- + check  $\rightarrow$  forward secrecy

# STS protocol (with explicit authentication)

- Station-to-Station protocol – forward secrecy  
(compromise of the long-term keys does not compromise the session keys established in previous protocol runs)

(Diffie) shared prime  $p$  and generator  $g$  of  $Z_p^*$

- 1. A  $\rightarrow$  B :  $ta = g^{ra} \text{ mod } p$
- 2. B  $\rightarrow$  A :  $tb = g^{rb} \text{ mod } p, \{\text{Sig}_B(tb, ta)\}K'$
- 3. A  $\rightarrow$  B :  $\{\text{Sig}_A(ta, tb)\}K'$       verify

$K = (ta)^{rb} \text{ mod } p = (tb)^{ra} \text{ mod } p$      $K' = f(K)$  ( $ra, rb$  – ephemeral keys)

(Lowe's attack)

- 1. A  $\rightarrow C_B$  :  $\text{Cert}(A), \text{Cert}(B), ta = g^{ra} \text{ mod } p$
- 1'. C  $\rightarrow$  B :  $\text{Cert}(C), \text{Cert}(B), ta = g^{ra} \text{ mod } p$
- 2'. B  $\rightarrow$  C :  $\text{Cert}(B), \text{Cert}(C), tb = g^{rb} \text{ mod } p, \{\text{Sig}_B(tb, ta)\}K'$
- 2.  $C_B \rightarrow$  A :  $\text{Cert}(B), \text{Cert}(A), tb = g^{rb} \text{ mod } p, \{\text{Sig}_B(tb, ta)\}K'$
- 3. A  $\rightarrow C_B$  :  $\text{Cert}(A), \text{Cert}(B), \{\text{Sig}_A(ta, tb)\}K'$



# STS protocol

modified (avoid Lowe's attack)

- 1. A  $\rightarrow$  B :  $ta = g^{ra} \text{ mod } p$
- 2. B  $\rightarrow$  A :  $tb = g^{rb} \text{ mod } p, \text{Sig}_B(tb, ta, A)$
- 3. A  $\rightarrow$  B :  $\text{Sig}_A(ta, tb, B)$       verify  
 $K = (ta)^{rb} \text{ mod } p = (tb)^{ra} \text{ mod } p$

STS protocol using MAC

- 1. A  $\rightarrow$  B :  $ta = g^{ra} \text{ mod } p$
- 2. B  $\rightarrow$  A :  $tb = g^{rb} \text{ mod } p, \text{Sig}_B(tb, ta), [tb, ta]K'$
- 3. A  $\rightarrow$  B :  $\text{Sig}_A(ta, tb), [ta, tb]K'$   
 $K = (ta)^{rb} \text{ mod } p = (tb)^{ra} \text{ mod } p$      $K' = f(K)$

# Oakley protocol (RFC 2412)

- aggressive mode

- 1. A → B:  $CK_A$ ,  $ta = g^{ra} \bmod p$ , list, A, B,  $Na$ ,  $Sig_A(A, B, Na, ta, list)$
- 2. B → A:  $CK_B$ ,  $CK_A$ ,  $tb = g^{rb} \bmod p$ , algo, B, A,  $Nb$ ,  $Na$ ,  
 $Sig_B(B, A, Nb, Na, tb, ta, algo)$  verify
- 3. A → B:  $CK_A$ ,  $CK_B$ ,  $ta$ , algo, A, B,  $Na$ ,  $Nb$ ,  
 $Sig_A(A, B, Na, Nb, ta, tb, algo)$  verify  
 $Z = (ta)^{rb} \bmod p = (tb)^{ra} \bmod p$      $K = [CK_A, CK_B, Z](Na, Nb)$

- conservative mode

- 1. A → B: OK                       $CK_A, CK_B$  - cookies (to mitigate DoS attacks)
- 2. B → A:  $CK_B$
- 3. A → B:  $CK_A, CK_B, ta = g^{ra} \bmod p$ , list
- 4. B → A:  $CK_B, CK_A, tb = g^{rb} \bmod p$ , algo                       $K_E = \#(Z)$
- 5. A → B:  $CK_A, CK_B, ta, \{A, B, \{|Na|\}Kb\}K_E$                        $K_M = \#(Na, Nb)$
- 6. B → A:  $CK_B, CK_A, \{\{|Nb, Na|\}Ka, B, A, [B, A, tb, ta, algo]K_M\}K_E$
- 7. A → B:  $CK_A, CK_B, \{[A, B, ta, tb, algo]K_M\}K_E$                       verify MAC  
 $Z = (ta)^{rb} \bmod p = (tb)^{ra} \bmod p$      $K = [CK_A, CK_B, Z](Na, Nb)$

# Oakley protocol with encryption

- aggressive mode

- 1. A → B:  $CK_A, ta = g^{ra} \bmod p, list, B, \{|A, B, Enc_B(Na)|\}_{KB'}$
- 2. B → A:  $CK_B, CK_A, tb = g^{rb} \bmod p, algo, \{|B, A, Nb|\}_{KA}, [B,A,Nb,Na,tb,ta,algo]_K$
- 3. A → B:  $CK_A, CK_B, [A, B, ta, tb, algo]_K$  verify MAC  
 $K = [CK_A, CK_B, (ta)^{rb} \bmod p = (tb)^{ra} \bmod p](Na, Nb)$

# SKEME protocol

- SKEME (for IPsec)

- 1. A → B:  $\{|A, Na|\}_{Kb}$ ,  $ta = g^{ra} \text{ mod } p$
- 2. B → A:  $\{|Nb|\}_{Ka}$ ,  $tb = g^{rb} \text{ mod } p$ ,  $[ta, tb, B, A]_{K'}$
- 3. A → B:  $[tb, ta, A, B]_{K'}$

$$K = (ta)^{rb} \text{ mod } p = (tb)^{ra} \text{ mod } p ; \quad K' = \#(Na, Nb)$$

- SKEME without DH

- 1. A → B:  $\{|A, Na|\}_{Kb}$ ,  $ra$
- 2. B → A:  $\{|Nb|\}_{Ka}$ ,  $rb$ ,  $[ra, rb, B, A]_{K'}$
- 3. A → B:  $[rb, ra, A, B]_{K'}$

$$K = [[rb, ra, A, B]_{K'}]_{K'} ; \quad K' = \#(Na, Nb)$$

# SKEME and IKE protocol

- IKE (Internet Key Exchange) main IPsec
    - 1. A -> B:  $CK_A$ , list  $CK_A, CK_B$  - cookies (to mitigate DoS attacks)
    - 2. B -> A:  $CK_B$ , algo
    - 3. A -> B:  $CK_A, CK_B, ta = g^{ra} \bmod p, Na$   $|Na|, |Nb| \in \langle 64, 2048 \rangle$
    - 4. B -> A:  $CK_A, CK_B, tb = g^{rb} \bmod p, Nb$
    - 5. A -> B :  $CK_A, CK_B, \{A, Sig_A([ta, tb, CK_A, CK_B, list, A]K')\}K$
    - 6. B -> A :  $CK_A, CK_B, \{B, Sig_B([tb, ta, CK_B, CK_A, list, B]K')\}K$
- $Z = (ta)^{rb} \bmod p = (tb)^{ra} \bmod p$   $K' = [Z]NaNb$   
 $K = SKDF(K')$  symmetric key derivation function

# SIGMA and IKEv2 protocol

- SIGMA-I (Krawczyk, Canetti Sign-and-MAC)

1. A -> B :  $ta = g^{ra} \text{ mod } p$

2. B -> A :  $tb = g^{rb} \text{ mod } p, \{B, \text{Sig}_B(ta, tb), [B]K_M\}_{K_E}$

3. A -> B :  $\{A, \text{Sig}_A(tb, ta), [A]K_M\}_{K_E}$

$Z = (ta)^{rb} \text{ mod } p = (tb)^{ra} \text{ mod } p \quad K_M = f_M(Z); K_E = f_E(Z)$

- IKEv2 (Internet Key Exchange) IPsec RFC 7296 (simplified)

## IKE\_INIT

• A -> B: list,  $ta = g^{ra} \text{ mod } p, Na \quad |Na|, |Nb| \in \langle 64, 2048 \rangle$

• B -> A: algo,  $tb = g^{rb} \text{ mod } p, Nb$

## IKE\_AUTH

• A -> B :  $\{A, \text{Sig}_A(\text{list}, ta, Na, Nb, [A]K_M)\}_{K_E}$  verify MAC

• B -> A :  $\{B, \text{Sig}_B(\text{algo}, tb, Nb, [B]K_M)\}_{K_E}$  verify MAC

$Z = (ta)^{rb} \text{ mod } p = (tb)^{ra} \text{ mod } p; K_M = [Z]NaNb; K_E = f(K_M); K = \text{SKDF}(K_M)$

CK – on demand (DoS)

# JFK – Just Fast Keying

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# ISO/IEC 11770-3 Key agreement

- 12 key agreement mechanisms
- analysed using the Scyther tool
- mechanism 11 (TLS handshake)
  - the RSA version has problems in authentication,
  - protocol does not provide forward secrecy



# TLS – Transport Layer Security

- 1995 SSL (Netscape Communications)
  - 1999 TLS 1.0 RFC 2246
  - 2006 TLS 1.1 RFC 4346
  - 2008 TLS 1.2 RFC 5246
  - 2018 TLS 1.3 RFC 8446
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- reliable transport protocol (over TCP)

# protocol model for TLS

- 0. A  $\rightarrow$  B: A, Na, Sid, Pa
- 1. B  $\rightarrow$  A: Nb, Sid, Pb
- 2. B  $\rightarrow$  A:  $\{|B, Kb|\}$ inv(Ks)
  - (Ks := pubkey of the certification authority – this is a certificate)
- 3. A  $\rightarrow$  B:  $\{|A, Ka|\}$ inv(Ks)
- 4. A  $\rightarrow$  B:  $\{|PMS|\}$ Kb
- 5. A  $\rightarrow$  B:  $\{| \#(Nb, B, PMS) |\}$ inv(Ka)
- 6. A  $\rightarrow$  B: {Finished}Keygen(A, Na, Nb, M)
  - where  $M = \text{PRF}(PMS, Na, Nb)$  (PseudoRandomFunction)
  - Finished = H(M, messages) for all messages 0 - 5
- 7. B  $\rightarrow$  A: {Finished}Keygen(B, Na, Nb, M)



# Simplified TLS

- 1. A -> B:  $N_a$
  - 2. B -> A:  $N_b$                       PMK – Pre-master key from A
  - 3. A -> B:  $\{|PMK|\}K_b, \text{Sig}_A(\text{Mess\_Seq1}), \{\text{Mess\_Seq2}\}K$
  - 4. B -> A:  $\{\text{Mess\_Seq3}\}K$                        $K = [N_a, N_b]PMK$   
Mess\_Seq1 =  $\#(N_a, N_b, \{|PMK|\}K_b) \dots$
- 
- simplified (based on Diffie-Hellman)
    - 1. A -> B:  $N_a$
    - 2. B -> A:  $N_b$                        $PMK = g^{a*b} \text{ mod } p$
    - 3. A -> B:  $\text{Sig}_A(\text{Mess\_Seq1}), \{\text{Mess\_Seq2}\}K$
    - 4. B -> A:  $\{\text{Mess\_Seq3}\}K$                        $K = [N_a, N_b]PMK$

# TLS with Perfect Forward Secrecy

- TLS with forward secrecy

- 1. A  $\rightarrow$  B:  $N_a$
- 2. B  $\rightarrow$  A:  $N_b, g^{r_b} \bmod p, \text{Sig}_B(N_a, N_b, p, g, A)$
- 3. A  $\rightarrow$  B:  $g^{r_a} \bmod p$

$Z = g^{r_a * r_b} \bmod p$       delete  $r_a, r_b$  (ephemeral keys)

MasterK =  $[N_a, N_b]Z$

SessK =  $\#(\text{MasterK expansion}, N_a, N_b)$

ECDHE – ephemeral DH with elliptic curve cryptography

PFS – perfect forward secrecy - previous session keys are not compromised even if the long term keys are

# KEM key encapsulation mechanisms

- encryption of new random number
- public  $K \in K_E$  , private  $K^{-1} \in K_D$
- encapsulation  $\text{Encap}_K() = (c, k)$   $k$  – new sym. key ,  $c \in C$
- decapsulation  $\text{Decap}_{K^{-1}}(c) = k$ 
  - A:  $(c_A, k_A) = \text{Encap}_B()$   
A  $\rightarrow$  B:  $c_A$
  - B:  $k_A = \text{Decap}_B(c_A)$ ,  $(c_B, k_B) = \text{Encap}_A()$   
B  $\rightarrow$  A:  $c_B$
  - $K = k_A, k_B$

# EKE – Encrypted Key Exchange

$\pi$  – shared password

- A  $\rightarrow$  B : A ,  $\{g^{ra} \bmod p\}\pi$
- B  $\rightarrow$  A :  $\{g^{rb} \bmod p\}\pi$  ,  $\{Nb\}K$
- A  $\rightarrow$  B :  $\{Na, Nb\}K$
- B  $\rightarrow$  A :  $\{Na\}K$

$ra, rb$  – ephemeral keys

$$K = g^{ra * rb} \bmod p$$

# PAK – Password Authenticated Key exchange

$\pi$  – shared password     $g$  – generator  $Z_p$      $p-1=q*t$

- $A \rightarrow B$  :  $(g^{ra} \bmod p) * H(A,B,\pi)^t = m$   
 $K = g^{ra * rb} \bmod p$
- $B \rightarrow A$  :  $g^{rb} \bmod p$ ,  $H'(A,B, m, g^{rb}, K)$   
 $K = \dots$
- $A \rightarrow B$  :  $H''(A,B, m, g^{rb}, K)$



# SPEKE - Secure Password Exponential Key Exchange

$\pi$  – shared password     $g$  – generator  $Z_p$      $p-1=q*t$   
 $P = \pi^2 \text{ mod } p$

• A  $\rightarrow$  B :     $P^{ra} \text{ mod } p$

$$K = P^{ra * rb} \text{ mod } p$$

• B  $\rightarrow$  A :     $P^{rb} \text{ mod } p$  ,  $H( P^{ra}, P^{rb}, K, \pi)$

$$K = \dots$$

• A  $\rightarrow$  B :     $H'( P^{ra}, P^{rb}, K, \pi)$

$$Z = H''(K)$$

# RSA-based protocols

$\pi$  – shared password,  $n = p * q$ ,  $d * e \bmod (p-1) * (q-1) = 1$

- A  $\rightarrow$  B :  $A, n, \{e\}\pi$
- B  $\rightarrow$  A :  $\{K^e \bmod n\}\pi$

