Efficient Broadcasting in Known Topology Radio Networks with Long-range Interference

František Galčík [*]
Institute of Computer Science
P.J. Šafárik University in Košice
040 01 Košice, Slovakia
Frantisek.Galcik@upjs.sk

Leszek Gąsieniec [†]
Department of Computer Science
University of Liverpool
Liverpool L69 7ZF, UK
L.A.Gasieniec@liv.ac.uk

Andrzej Lingas [‡]
Department of Computer Science
Lund University
S-221 00 Lund, Sweden
Andrzej.Lingas@cs.lth.se

ABSTRACT

We study broadcasting (one-to-all communication) in known topology radio networks modeled by graphs, where the interference range of a node is likely to exceed its transmission range. In this model, if two nodes are connected by a *transmission edge* they can communicate directly. On the other hand, if two nodes are connected by an *interference edge* their transmissions disable recipience of one another. For a network G, we term the smallest integer d, s.t., for any interference edge e there exists a simple path formed of at most d transmission edges connecting the endpoints of e as its *interference distance* d_I. In this model the schedule of transmissions is precomputed in advance based on full knowledge about the size and the topology (including location of transmission and interference edges) of the network. We are interested in the design of fast broadcasting schedules that are energy efficient, i.e., based on limited number of transmissions at each node.

In what follows we assume that n stands for the number of nodes, D_T is the diameter of the subnetwork induced by the transmission edges, and Δ refers to the maximum *combined degree* (formed of transmission and interference edges) of the network. We contribute the following new results:

(1) We prove that even for networks with the interference distance $d_I = 2$ any broadcasting schedule requires at least $D_T + \Omega\left(\Delta \cdot \frac{\log n}{\log \Delta}\right)$ rounds.

(2) We also provide for networks modeled by bipartite graphs an algorithm that computes 1-shot (each node is allowed to transmit at most once) broadcasting schedules of length $O(\Delta \cdot \log n)$.

Note that in this case the length of the broadcasting schedule is independent of the interference distance of the network.

(3) The **main result** of the paper is an algorithm that computes a 1-shot broadcasting schedule of length at most $4 \cdot D_T + O(\Delta \cdot d_I \cdot \log^4 n)$ for networks with arbitrary topology.

Note that in view of the lower bound from (1) the broadcast schedule is almost optimal for d_I polylogarithmic in n. Note also that by applying our algorithm to radio networks with no interference edges the time of the broadcasting schedule from [10] is improved in graphs with $\Delta = o(\frac{\sqrt{n}}{\log^4 n})$. The 1-shot broadcasting algorithm proposed in [10] relies heavily on the concept of *internal ranks* that impose currently an $\Omega(\sqrt{n})$-time bottleneck in the broadcasting schedule.

Categories and Subject Descriptors

F.2.2 [**Analysis of Algorithms and Problem Complexity**]: Nonnumerical Algorithms and Problems—*Sequencing and scheduling*; G.2.3 [**Discrete Mathematics**]: Applications

General Terms

Algorithms, Theory

Keywords

Radio Networks, Broadcasting, Long-range Interference

1. INTRODUCTION

A radio network is a collection of radio devices, referred to as *nodes*, that are able to communicate by transmitting and receiving radio signals. The transmitted radio signal contains an encoded message. Communication in the network is *synchronous* and all nodes operate on the same radio frequency. The network nodes operate in discrete steps (time slots) called also *rounds*. During each round, a node can be either in the *transmitter* or in the *receiver* mode. The most

[*]Research of the author is supported by the Slovak Grant Agency for Science (VEGA) under contract 1/0035/09

[†]Research of the author is supported in part by Royal Society International Joint Projects IJP - 2007/R1 & IJP - 2006/R2

[‡]Research of the author is supported in part by VR grants 621-2005-4085 and 621-2008-4649

challenging task in radio communication is coping with interference caused by simultaneous transmissions. If two or more radio signals of an appropriate intensity reach simultaneously a receiving node, it hears only the interference noise. The intensity and quality of signals transmitted by network nodes vary. They depend, e.g., on the distance from the transmitting node and properties of the surrounding environment. In order to model the communication environment more accurately, we introduce a radio network model with *long-range interference*.

In this model a network is represented by an undirected (symmetric) graph $G = (V, E = E_T \cup E_I)$ called an *interference reachability graph* IRG. The set of vertices V of the IRG, where $|V| = n$, corresponds to the network nodes. For each node $v \in V$, we define two ranges (sets). A *transmission range* $R_T(v)$ contains nodes that can be reached directly from v, i.e., where the transmitted signal has the quality that allows its decoding. On the contrary, an *interference range* $R_I(v)$ contains nodes at which a signal transmitted from v causes interference either on its own or together with other simultaneously incoming signals. Note that, if the transmitted signal is strong enough to be decoded, it is also strong enough to interfere, i.e., $R_T(v) \subseteq R_I(v)$. Also, if $w \in R_I(v) \setminus R_T(v)$, any transmission from v always causes interference at the node w. The set of edges E in the IRG G is partitioned into two disjoint groups: a set of *transmission edges* E_T and a set of *interference edges* E_I. An edge $(u, v) \in E_T$ if and only if the node v is located in the transmission range of the node u, i.e., $(u, v) \in E_T \Leftrightarrow v \in R_T(u)$. In this case the node u is called a *transmission neighbor* of the node v. The induced subgraph G_T on the set of transmission edges is called a *transmission subgraph*. We also denote by D_T the diameter of G_T, by $ecc_T(v)$ the eccentricity (distance to the most remote node) of v and by $deg_T(v)$ the degree of v in G_T. An edge $(u, v) \in E_I$ if and only if the node v belongs to the interference range of the node u but not to its transmission range, i.e., $(u, v) \in E_I \Leftrightarrow v \in R_I(u) \setminus R_T(u)$. In this case the node u is called an *interference neighbor* of the node v. We also denote by $deg_I(v)$ the number of incident interference edges to v in IRG and we say that a network (represented by an IRG) has the *interference distance* d_I if d_I is the smallest integer, s.t., for any interference edge $e \in E_I$ there exists a path along at most d_I transmission edges connecting the endpoints of e. Any path along transmission edges is called a *transmission path*. We denote by $d_T(u, v)$ the smallest integer, s.t., there is a transmission path with $d_T(u, v)$ edges between the nodes u and v. Finally, the maximum (combined) degree of IRG is denoted by Δ.

One of the most fundamental communication primitives in networks is *broadcasting*. The main purpose of broadcasting is to distribute a *message*, from a distinguished node, called the *source* s, to all other nodes nodes in the network. During the communication process the immediate neighbors of the source can be reached directly from s while the remote nodes have to be informed via intermediate connections imposing a *multihop communication protocol*. In this paper we focus on the *time complexity* of radio broadcasting, i.e., a number of rounds required by a communication protocol to accomplish the communication task. Moreover, since radio network devices are very often powered by batteries with bounded capacity, our other high priority are protocols characterized by limited *energy consumption*. In fact the main emphasis in this paper is on 1-shot broadcasting protocols in which each node is allowed to transmit at most once, see earlier work in [2, 10]. We assume that the transmission subgraph G_T is connected. This is to guarantee that the broadcast message can be delivered from an arbitrarily chosen source node to any other node in the network.

1.1 Related work

The greatest effort in the distributed algorithms community devoted to efficient communication in radio networks refers to the *packet radio network model* that was introduced by Chlamtac and Kutten in [4]. In this model one assumes that $R_T(v) = R_I(v)$, i.e., the transmission and interference ranges of each node are the same. The study of efficient broadcasting in the radio networks with known topology was initiated by Chlamtac and Weinstein in [5]. The authors presented a deterministic polynomial-time algorithm that generates a broadcasting schedule of length $O(D \cdot \log^2 n)$, for any network with n nodes and the diameter D. This result was later complemented by Alon *et al.* in [1], where the lower bound $\Omega(\log^2 n)$ on the length of a broadcasting schedule was proved for a family of graphs with the eccentricity of the source equal to 2. Later in [8], the authors proposed a method improving the time of the broadcasting to $O(D + \log^5 n)$ rounds. The method is based on partitioning of an underlying communication graph into clusters with smaller diameter and applying broadcasting schedules produced by known algorithms in each cluster separately. An improvement of this method was later presented in [7]. Applying a deterministic algorithm from [14], which produces a broadcasting schedule of a length $O(D \cdot \log n + \log^2 n)$, method from [7] computes a broadcasting schedule of a length $O(D + \log^4 n)$ rounds. Note that the algorithm for arbitrary networks, that is presented in this paper, adopts and generalises these methods (from [7] and [8]) to include a long-range interference. Recently, the result from [7] was further improved in [11] to $D + O(\log^3 n)$. Finally in [15], the authors proposed an algorithm producing a radio broadcasting schedule of the asymptotically optimal length $O(D + \log^2 n)$. A detailed survey of known results concerning time-efficient broadcasting (centralized and distributed with only local knowledge) can be found in [18] or [19]. Time efficient broadcasting with restriction on energy consumption (the number of transmissions per node) was first investigated in unknown graphs by Berenbrink *et al.* in [2] and later in known graphs by Gąsieniec *et al.* in [10], where the notion of $k-$shot protocols was introduced. In particular, in [10] one can find $1-$shot broadcast schedules of lengths $O(\sqrt{n})$ and $D + O(\sqrt{n} \cdot \log n)$ for bipartite and arbitrary graphs respectively. The results in this paper should be seen as improved radio broadcast schedules, in relation to [10], for graphs with $\Delta = o(\frac{\sqrt{n}}{\log^4 n})$.

When transmission and interference ranges differ, more particular assumptions about model characteristic make an immediate impact on communication. Moreover, it appears that there is no widely accepted model of radio networks with long-range interference. Radio networks are very often modelled geometrically, where transmission and interference ranges are defined in terms of Euclidean distances, and designed algorithms are probabilistic. Among the others, there is a model proposed by Gupta and Kumar in [13] and known as *Signal-to-Interference-plus-Noise-Ratio (SINR)* that received recently more attention. The SINR model is a more complex model since in which each transmission is given a power and we assume a distance and noise dependent power loss. A transmission is deemed to be successful if the signal at a destination is stronger than some specified threshold. Broadcasting in SINR model was investigated in [17]. Also more recent study on characteristics of efficient local broadcasting in the SINR model can be found in Goussevskaia *et al.* in [12], and the study on connectivity and interference in log-normal shadowing radio propagation model by Muetze et al. in [16].

In this paper, we focus on a variant (with long-range interference) of more classical graph-based model of radio networks from [4]. Such variants were recently investigated for the gathering problem by Bermond et al. in [3] and for broadcasting in graphs with locally bounded density of interference edges by Galčík in [9]. While we adopt here the model of communication from [9], this time we focus our attention on *interference distance* that reflects more accurately the impact of long-range interference on communication in radio networks.

2. LOWER BOUND

THEOREM 1. *There exists an IRG $G = (V, E_T \cup E_I)$ with n nodes, $d_I(G) = 2$, an even maximal degree $\Delta \geq 4$, and the eccentricity of the source in the transmission subgraph $ecc_T(s)$ satisfying $72 \leq 6 \cdot \frac{\log n}{\log \Delta} \leq ecc_T(s) \leq n/2$, s.t., any broadcast schedule requires $ecc_T(s) + \Omega\left(\Delta \cdot \frac{\log n}{\log \Delta}\right)$ rounds.*

PROOF. Consider a simple network structure that contributes a $(\Delta/2-1)$-round slowdown to the time complexity of information dissemination. Let $G' = (V', E'_T \cup E'_I)$ be an IRG such that

- $V' = \{s\} \cup V_a \cup V_b \cup V_c$, where $V_a = \{a_1, \ldots, a_m\}$, $V_b = \{b_1, \ldots, b_m\}$, and $V_c = \{c_1, \ldots, c_m\}$
- $E'_T = \{(s, a_i), (a_i, b_i), (b_i, c_i) | 1 \leq i \leq m\} \cup \{(a_i, a_j) | 1 \leq i \neq j \leq m\}$
- $E'_I = \{(a_i, b_j) | 1 \leq i \neq j \leq m\}$.

Note that the subgraph induced by the nodes in V_a is a complete graph. It follows that $d_T(a_i, b_j) = 2$, for any i and j ($i \neq j$). Hence, $d_I(G') = 2$. One can observe that any radio broadcasting schedule with the node s as the source requires at least $m + 2$ rounds to be accomplished. And indeed, all nodes in V_a are informed after the first round, when the source s transmits. However, if two or more nodes in V_a transmit simultaneously, no node in V_b receives a message due to presence of interference edges. Hence, it is not possible to inform all nodes in V_b in less than m consequent rounds. Finally, we need one extra round to inform all nodes in V_c. Therefore, the total time required is $1+m+1$ rounds. In comparison to a naive 3-round broadcasting for the case without interference edges, we obtain an $(m-1)$-round slowdown.

Further, we extend this argument to networks with a larger eccentricity of the source. Let $T_{r,h}$ be a perfect r-nary tree of height h. All internal nodes of $T_{r,h}$ have exactly r children, i.e., their degree is $r+1$. The total number of nodes in the tree $T_{r,h}$ is $\frac{r^{h+1}-1}{r-1}$. We turn a tree $T_{r,h}$ into an IRG $T'_{r,h}$ as follows. First, we insert new nodes v'_i and v''_i, including corresponding edges, between each internal node v and its

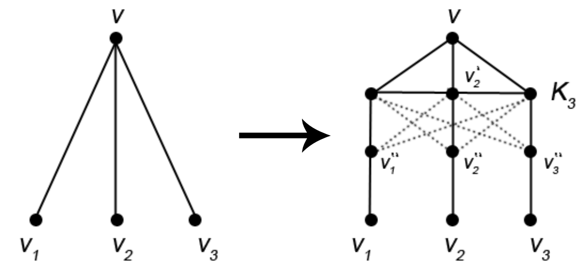

Figure 1: Transformation of a node v and its children v_1, v_2, v_3 in $T_{3,h}$ to a set of nodes and edges in $T'_{3,h}$.

child v_i, for $i = 1, \ldots, r$. More precisely, the edge (v, v_i) in $T_{r,h}$ is replaced with transmission edges (v, v'_i), (v'_i, v''_i), and (v''_i, v_i) in $T'_{r,h}$. Next, we add an interference edge (v'_i, v''_j) to $T'_{r,h}$, for each $1 \leq i \neq j \leq r$. These newly added interference edges contribute to the slowdown, because neither pair of nodes v'_i and v'_j is allowed to transmit simultaneously in order to inform nodes v''_i and v''_j. Finally, we add a transmission edge (v'_i, v'_j) to $T'_{r,h}$, for each $1 \leq i \neq j \leq r$. These edges do not improve the communication process. In fact, they impose the interference distance 2 in the IRG. A similar argument (as we used for the IRG G') implies that in any given radio broadcasting schedule (with s as the source), each internal node $v \in T'_{r,h}$ has a descendant node v_i (a child in $T_{r,h}$) which does not get the message earlier than $r + 2$ rounds after v becomes informed. Hence, using inductive argument, any radio broadcasting schedule in $T'_{r,h}$ requires $3 \cdot h + h \cdot (r - 1)$ transmission rounds. The resulting IRG $T'_{r,h}$ consists of $3 \cdot \frac{r^{h+1}-1}{r-1} - 2$ nodes and the eccentricity of s is $3 \cdot h$. In order to change the eccentricity of the source to more arbitrary value $ecc_T(s)$, we add a simple path of length $ecc_T(s) - 3 \cdot h$ to $T'_{r,h}$. One end of the path gets connected with the current location of s in $T'_{r,h}$ while the other end becomes the new source s. This new network, denoted by $T'_{r,h,ecc_T(s)}$, consists of $n(h) = 3 \cdot \frac{r^{h+1}-1}{r-1} - 2 + ecc_T(s) - 3 \cdot h$ nodes. In $T'_{r,h,ecc_T(s)}$, any radio broadcasting schedule requires at least $ecc_T(s) + h \cdot (r - 1)$ rounds.

In the following, consider only IRGs $T'_{r,h,ecc_T(s)}$, where $r = \Delta/2$. Let h_m be an integer such that $n(h_m) \leq n < n(h_m + 1)$. Now assuming that $ecc_T(s) \geq 6 \cdot \frac{\log n}{\log \Delta}$, one can show that for any $h \geq 2$ satisfying the inequality $n(h) \leq n$ we get $3 \cdot h \leq ecc_T(s)$. Also, the inequalities $n(h_m + 1) > n$ and $ecc_T(s) \leq n/2$ imply that $h_m \geq \frac{\log n}{6 \cdot \log \Delta}$. Since $36 \cdot h_m \geq 6 \cdot \frac{\log n}{\log \Delta} \geq 72 = 36 \cdot 2$, we get $h_m \geq 2$. It implies that $3 \cdot h_m \leq ecc_T(s)$. Thus, the construction of $T'_{\Delta/2,h_m,ecc_T(s)}$ is correct. The IRG $T'_{\Delta/2,h_m,ecc_T(s)}$ is an IRG with $n(h_m)$ nodes, the eccentricity of the source $ecc_T(s)$, the maximum degree Δ, $d_I(T'_{\Delta/2,h_m,ecc_T(s)}) = 2$, and the broadcasting time at least $ecc_T(s) + h_m \cdot (\Delta/2 - 1) \geq ecc_T(s) + \frac{\log n}{6 \cdot \log \Delta} \cdot (\Delta/2 - 1) = ecc_T(s) + \Omega\left(\Delta \cdot \frac{\log n}{\log \Delta}\right)$. It remains to transform the IRG $T'_{\Delta/2,h_m,ecc_T(s)}$ to an IRG G with n nodes and satisfying all required properties. Observe that only a relatively small number of nodes is not in $T'_{\Delta/2,h_m,ecc_T(s)}$. Indeed, we can bound from above the number of remaining network nodes by $n - n(h_m) \leq n(h_m + 1) - n(h_m) \leq (\Delta/2)^{h_m+3}$.

The number of nodes in a $(\Delta-1)$-nary tree with the height at most $3 \cdot h_m - 1$ is bounded from above by $\frac{(\Delta-1)^{3 \cdot h_m}-1}{\Delta-2}$.

For $h_m \geq 2$, this expression gives a value greater than $(\Delta/2)^{h_m+3}$, which is an upper bound on the number of remaining nodes. Since the source of the IRG $T'_{\Delta/2,h_m,ecc_T(s)}$ has degree at most $\Delta - 1$ (it has at most $\Delta/2$ children and no parent), we can arrange the remaining $n - n(h_m)$ nodes into a complete $(\Delta-1)$-nary tree rooted in a newly created child of the source of $T'_{\Delta/2,h_m,ecc_T(s)}$. The newly constructed IRG constitutes G from the thesis of the theorem. Recall that $ecc_T(s) \geq 3 \cdot h_m$ and the height of the complete tree formed of the remaining nodes is at most $3 \cdot h_m - 1$. Therefore, broadcasting in the tree of remaining nodes can be completed separately and independently from any broadcasting process in $T'_{\Delta/2,h_m,ecc_T(s)}$ during at most $ecc_T(s)$ rounds after first transmission of the source. The thesis of the theorem follows. □

3. INFORMATION DISSEMINATION IN BIPARTITE NETWORKS

A bipartite communication network with one part formed by informed nodes and the other formed by their uninformed neighbors is an important combinatorial structure used in the context of broadcasting. In this section we show how to explore this concept in radio networks with long-range interference. We start with an algorithm that generates a fast 1-shot schedule of transmissions allowing to inform all nodes in the uninformed part.

The following lemma can be also seen as a consequence of the Markov innequality.

LEMMA 2. *For any set of natural numbers $\{d_1, d_2, \ldots, d_n\}$, s.t., $d_i \geq 0$ and $\sum_{i=1}^{n} d_i < h \cdot n$, it holds that $|\{d_i | d_i \leq 2 \cdot h\}| \geq n/2$.*

PROOF. Assume contrarily that $|\{d_i | d_i \leq 2 \cdot h\}| \leq n/2$ which is equivalent to $|\{d_i | d_i > 2 \cdot h\}| \geq n/2$. This implies $\sum_{i=1}^{n} d_i > 2 \cdot h \cdot \frac{n}{2}$ which contradicts one of the assumptions of the lemma. □

THEOREM 3. *Let $G = (V_S \cup V_R, E_T \cup E_I)$ be a bipartite IRG. Assume that all nodes in V_S are informed, i.e., they know the source message, and the nodes in V_R are uninformed. If $deg_T(v) \geq 1$, for all $v \in V_R$, there is a linear time algorithm that generates a 1-shot schedule of transmissions informing all nodes in V_R. The length of the schedule is $O(\Delta \cdot \log |V_S|)$ rounds.*

PROOF. Without loss of generality, we may assume that each node $v \in V_R$ is incident to exactly one transmission edge (i.e., $deg_T(v) = 1$) and each node $v \in V_S$ is incident to at least one transmission edge (i.e., $deg_T(v) \geq 1$). And indeed, if $deg_T(v) > 1$ for a node $v \in V_R$, we keep one of its incident transmission edges and consider all other incident transmission edges as interference edges. Furthermore, we remove each node $v \in V_S$ such that $deg_T(v) = 0$. These modifications do not improve chances of nodes in V_R to be informed earlier. Thus, in view of the transmission subgraph, the graph G is a collection of disjoint stars of transmission edges with centers in V_S and terminal nodes in V_R. Each interference edge connects the center of a star with some terminal node of another star. We define the sets of nodes and edges in an undirected graph G_c as follows:

- $V(G_c) = V_S$,
- $E(G_c) = \{(u,v) | \exists w \in V_R, (u,w) \in E_T \wedge (v,w) \in E_I\}$.

The graph G_c is a graph whose nodes correspond to stars, where two nodes are connected by an edge in G_c if and only if there is an interference edge joining the center of one star and a terminal node of the other. Observe that each edge in G_c corresponds to a path of the length 2 in G. The path consists of an interference edge and a transmission edge. Since $deg_T(v) = 1$, for each $v \in V_R$, it follows that an interference edge in G can introduce at most one new edge to G_c. Hence, $|E(G_c)| \leq |E_I| < |E(G)| \leq \Delta \cdot |V_S|$. Denote the degree of a node $v \in V_S$ in the graph G_c by $deg_c(v)$. It follows that $\sum_{v \in V_S} deg_c(v) = 2 \cdot |E(G_c)| < 2 \cdot \Delta \cdot |V_S|$. Applying $h = 2 \cdot \Delta$ to Lemma 2 we get $|\{v \in V_S | deg_c(v) \leq 4 \cdot \Delta\}| \geq |V_S|/2$. I.e., at least half of the nodes in V_S have their degree in G_c lower than $4 \cdot \Delta$. Now, we remove all nodes with the degree strictly greater than $4 \cdot \Delta$ from G_c obtaining G'_c. Since the maximal degree in G'_c is less than $4 \cdot \Delta$, we can color efficiently [20] the nodes in G'_c using at most $4 \cdot \Delta + 1$ colors. Finally, observe that if the nodes in $V(G'_c)$ transmit in rounds corresponding to the assigned colors, all their transmission neighbors will be successfully informed. And indeed, if there is a node $w \in V_R$ and two nodes $u, v \in V(G'_c)$, s.t., $(u,w) \in E_T$ and $(v,w) \in E_I$, then $(u,v) \in E(G'_c)$. It follows that $color(u) \neq color(v)$, i.e., the nodes u and v transmit in different rounds. Summarising , at least half of the nodes in V_S inform all their transmission neighbors during at most $4 \cdot \Delta + 1$ rounds. These nodes can be removed together with already informed nodes in V_R from the graph G. And iterating this process at most $\log |V_S|$ times, we obtain a $O(\Delta \cdot \log |V_S|)$-round schedule of transmissions informing all nodes in V_R.

Finally, note that each node in V_S is prompted to transmit at most once in the schedule. This is due to the fact that each node in V_S acts as a node of G'_c in at most one iteration of the algorithm and during each iteration, each node transmits at most once. Thus we obtained a 1-shot schedule of transmissions. □

OBSERVATION 4. *Since an IRG without interference edges corresponds to a reachability graph in the standard graph model, the presented algorithm can be used to generate 1-shot schedules for radio networks modelled by the standard graph model. Produced $O(\Delta \cdot \log n)$-round schedules is an alternative to $O(\sqrt{n})$-time schedules generated by the algorithm in [11].*

4. FAST BROADCASTING IN ARBITRARY NETWORKS WITH LONG-RANGE INTERFERENCE

Overview of the algorithm. Our new algorithm generating broadcast schedules adopts an approach in which the set of transmissions is divided into *fast* and *slow* transmissions. This universal paradigm is used with various modifications in almost all algorithms generating broadcast schedules in known radio networks. In this approach, a source message is disseminated along branches of a BFS tree-like subnetwork using fast transmissions pipelined along selected simple paths and a limited number of slow transmissions based on propagation of information in bipartite graphs. In our approach we utilise, with required modifications, the clustering mechanism presented by Gaber and Mansour in [8], however, the presence of interference edges imposes certain structural changes. A notable difference is that the constructed clusters do not form a connected subgraph, although there is a short transmission path between any two nodes of the same cluster. Another difference refers to the lack of short transmission paths between the clusters of the same colour (rather than the lack of direct edges) that enables efficient dissemination mechanism in radio networks with long-range interference. Finally, whereas fast transmissions are realized on the basis of the tree-like communication subnetwork, slow transmission are executed in successive stages in the form of a flooding mechanism where information is disseminated between collections of informed and neighboring uninformed nodes of dynamically evolving bipartite IRG. Note that the slow and fast transmissions despite being treated separately, they must be neatly coordinated to enable energy efficient 1-shot communication protocol.

4.1 Construction of clusters

Layers and super-layers. The broadcast schedule designed for IRGs utilises a decomposition of the input graph G into BFS layers, super-layers and a collection of overlapping clusters constructed on the basis of the transmission subgraph in G.

DEFINITION 5. *Let $G_T = (V, E_T)$ be the transmission subgraph of an undirected IRG $G = (V, E_T \cup E_I)$ with the source node $s \in V$. The i-th BFS layer of G is defined as $L_i = \{v \in V | d_T(s, v) = i\}$.*

The input IRG G consists of $ecc_T(s) + 1$ layers where $ecc_T(s) = max\{d_T(s, v) | v \in V(G)\}$ is defined as the *eccentricity* of the source node s in the transmission subgraph of G. In what follows, x denotes a parameter whose value will be determined later in Lemma 15.

DEFINITION 6. *For each $i = k \cdot x \le ecc_T(s) + 1$, the layer L_i of G is called the k-th inter-communication layer. The union of layers $\mathcal{L}_k = \bigcup\{L_i | k \cdot x \le i \le min(ecc_T(s), (k+1) \cdot x)\}$ forms the k-th super-layer of the IRG G. The layers $L_{k \cdot x}$ and $L_{min(ecc_T(s),(k+1)\cdot x)}$ form the highest and the lowest layers in the k-th super-layer respectively.*

The last definition implies that each super-layer consists of $x + 1$ layers and exactly two inter-communication layers: the highest and the lowest layer of the super-layer. The exemption is the most distant super-layer that might have smaller number of layers.

Clusters, pre-clusters and pre-cluster graphs. The clusters are built in each super-layer of G independently. Each cluster is a union of carefully crafted *pre-clusters* defined as follows. For each node $v \in L_{k \cdot x} \subseteq \mathcal{L}_k$ we define a pre-cluster as a set of nodes $S(v) = \{u | u \in L_{k \cdot x + i} \wedge d_T(v, u) = i \wedge 0 \le i \le x\}$. Note that the pre-cluster $S(v)$ contains all nodes in the same super-layer that are reachable from v along transmission edges used in the direction away from the source. The node v is called the *top node* of $S(v)$.

We define an undirected *pre-cluster graph* G_k for a given super-layer $\mathcal{L}_k$ as follows:

- $V(G_k) = L_{k \cdot x}$,
- $E(G_k) = \{(u, v) | \exists u', v' : u' \in S(u) \wedge v' \in S(v) \wedge d_T(u', v') \le d_I(G) + 1\}$.

The nodes in G_k correspond to the top nodes of pre-clusters located in the $k-$th super-layer. For any pair of nodes $u, v \in V(G_k)$ there is an edge connecting them in $E(G_k)$ if and only if there is a transmission path of length not exceeding $d_I(G)+1$ that connects some node in $S(u)$ with some node in $S(v)$. The structure of G_k guaranties, e.g., that there are no interference edges between nodes from different pre-clusters whose top nodes are not connected by an edge in G_k. In fact, the structure of G_k implies also several other powerful properties that are summarised in Lemma 8.

The system of clusters. The following theorem is due to Gaber and Mansour, see [8].

THEOREM 7. *Let $G = (V, E)$ be an undirected graph. There exists a clustering $\mathcal{C} = \{C_1, \ldots, C_r\}$ of G with the following properties:*

1. *$V(G) = \bigcup\{C_i | 1 \le i \le r\}$*
2. *$G[C_i]$ is a connected subgraph of G with a diameter at most $2 \cdot \log |V(G)|$*
3. *There is a proper coloring of clusters with at most $\lceil \log |V(G)| \rceil$ colors. The proper coloring of clusters satisfies*

$$(u = v \vee (u, v) \in E(G)) \wedge u \in C_i \wedge v \in C_j \wedge i \ne j \Rightarrow color(C_i) \ne color(C_j).$$

4. *$|\mathcal{C}| \le |V(G)|$*

In addition, the clustering can be constructed in $O(|E(G)| \cdot \log |V(G)|)$ time.

Let $\mathcal{C} = \{C_1, \ldots, C_r\}$ be a clustering of the pre-cluster graph G_k obtained by application of the clustering procedure from [8]. A clustering $\mathcal{C}^{(k)} = \{C_1^{(k)}, \ldots, C_r^{(k)}\}$ of pre-clusters in the k-th super-layer $\mathcal{L}_k$ is defined as $C_i^{(k)} = \bigcup\{S(v) | v \in L_{k \cdot x} \wedge v \in C_i\}$. Note that a color assigned to a cluster $C_i^{(k)}$ is the same as the color assigned to the cluster C_i in the clustering $\mathcal{C}$ of G_k. It also follows that coloring of $\mathcal{C}^{(k)}$ uses at most $\lceil \log |V(G_k)| \rceil \le \lceil \log |\mathcal{L}_k| \rceil$ colors.

LEMMA 8. *The clustering $\mathcal{C}^{(k)}$ has the following properties:*

1. *$\mathcal{L}_k = \bigcup\{C_i^{(k)} | 1 \le i \le |\mathcal{C}^{(k)}|\}$*
2. *$\forall u, v \in V(G)$:*

$$u, v \in C_i^{(k)} \Rightarrow d_T(u, v) \le 6 \cdot (d_I(G) + x) \cdot \log |\mathcal{L}_k|$$

3. *$\forall u, v, w \in V(G)$:*

$$u \in C_i^{(k)} \wedge v \in C_j^{(k)} \wedge i \ne j \wedge$$
$$color(C_i^{(k)}) = color(C_j^{(k)}) \wedge (u, w) \in E_T$$
$$\Rightarrow (v, w) \notin E_T \cup E_I$$

4. *$|\mathcal{C}^{(k)}| \le |L_{k \cdot x}|$*

PROOF. The property (1) is a consequence of property (1) in Theorem 7 and the definition of $\mathcal{C}^{(k)}$.

The property (2) says that for any pair of nodes $u, v \in C_i^{(k)}$ there exists a transmission path in G of a length less than $6 \cdot (d_I(G) + x) \cdot \log |\mathcal{L}_k|$. Note that this transmission path can go through nodes located outside of the cluster $C_i^{(k)}$. Hence, any two cluster nodes are connected in G_T, but not necessary in $G[C_i^{(k)}]$, i.e., the cluster $C_i^{(k)}$ can be disconnected. From the definition of $C_i^{(k)}$ one can conclude that there are two nodes $u_h, v_h \in C_i$, s.t., $u \in S(u_h)$ and $v \in S(v_h)$. Recall that C_i is a cluster defined on the pre-cluster graph G_k that determines the content of the cluster $C_i^{(k)}$. The property (2) of Theorem 7 implies that there exists a path $P = (u_h = w_1, w_2, \ldots, w_t = v_h)$ connecting nodes u_h and v_h in the pre-cluster graph G_k. Moreover, this property implies that the length of P is at most $2 \cdot \log |V(G_k)| = 2 \cdot \log |\mathcal{L}_k|$. From the definition of G_k one can conclude that there is a path of a length at most $d_I(G) + 1$ from a node in $S(w_i)$ to a node in $S(w_{i+1})$ in the transmission subgraph G_T, for any $i = 1, \ldots, t-1$. We denote this path by $P(w_i, w_{i+1})$, its first node by w_i^F and the last node by w_{i+1}^L. Since $w_i^F \in S(w_i)$, we get $d_T(w_i, w_i^F) \leq x$. Similarly, from $w_{i+1}^L \in S(w_{i+1})$ we obtain $d_T(w_{i+1}, w_{i+1}^L) \leq x$. Hence, there is a walk from w_i to w_{i+1} in G_T consisting of three parts: a path from w_i to w_i^F, the path $P(w_i, w_{i+1})$ from w_i^F to w_{i+1}^L, and a path from w_{i+1}^L to w_{i+1}. The total length of the walk is at most $x + (d_I(G)+1) + x$. Now, we are ready to construct a walk from u to v in G_T. The walk starts with a path from u to $u_h = w_1$. Further, it continues by a sequence of walks from w_i to w_{i+1}, for all $i = 1, \ldots, t-1$. The walk concludes with a path from $w_t = v_h$ to v. The total length of the constructed walk is at most $x + 2 \cdot \log |\mathcal{L}_k| \cdot (x + (d_I(G) + 1) + x) + x \leq 6 \cdot (d_I(G) + x) \cdot \log |\mathcal{L}_k|$. Hence, there exists a transmission path in G from u to v of length not exceeding $6 \cdot (d_I(G) + x) \cdot \log |\mathcal{L}_k|$.

The property (3) is proved by showing that there does not exist a transmission path in G of length smaller than $d_I(G) + 1$ that joins two nodes u, v in two different clusters with the same color. I.e., we must show that $d_T(u, v) > d_I(G) + 1$. If there is no transmission path from u to v, $d_T(u, v) = \infty$. And indeed, for any node $w \in V(G)$ connected to u by a transmission edge, i.e., $(u, w) \in E_T$, the inequality $d_T(u, v) > d_I(G) + 1 \geq 2$ implies that $d_T(w, v) > d_I(G)$. Hence, due to the definition of $d_I(G)$, we get that $(w, v) \notin E_T \cup E_I$. We show now that $d_T(u, v) > d_I(G) + 1$. Assume opposite, i.e., a transmission path from u to v exists and $d_T(u, v) \leq d_I(G) + 1$. Since $u \in C_i^{(k)}$, there is a node $u_h \in L_{k \cdot x}$, s.t., $u \in S(u_h)$ and $u_h \in C_i$, where C_i is one of the clusters in G_k. Similarly, $v \in C_j^{(k)}$ implies that there is a node $v_h \in L_{k \cdot x}$, s.t., $v \in S(v_h)$ and $v_h \in C_j$. Recall that we assumed $d_T(u, v) \leq d_I(G) + 1$. From the definition of G_k, it follows that $(u_h, v_h) \in E(G_k)$ and from the clustering construction we know that $color(C_i) = color(C_i^{(k)}) = color(C_j^{(k)}) = color(C_j)$. However, this leads to a contradiction since the coloring of clusters in G_k is a proper coloring. I.e., $(u_h, v_h) \in E(G_k)$ implies that $color(C_i) \neq color(C_j)$.

Finally, the property (4) follows directly from the property (4) in Theorem 7. □

Observe that the clustering $\mathcal{C}_k$ does not contain necessarily only internally connected clusters. In other words, a transmission path connecting two nodes of the same cluster can traverse through nodes outside of pre-clusters contributing to the cluster. On the other hand, the property (2) in Lemma 8 implies that the nodes of each cluster in $\mathcal{C}_k$ are connected in the graph G by relatively short transmission paths. Note that one can modify the clustering mechanism of Theorem 7 to obtain a clustering $\mathcal{C}_k$ that consists of internally connected clusters. The key idea of the modification lies in a way, how pre-clusters are gradually introduced to the currently constructed cluster. Instead of adding all unused neighboring pre-clusters of the currently constructed cluster, we insert unused pre-clusters containing at least one node which is connected by a transmission path of a length at most $d_I(G) + 1$ to some node of the currently constructed cluster.

4.2 Construction of ranked trees of clusters

The tree of clusters defines a parent-child relationship between clusters in neighboring super-layers and vice-versa. It is built in consecutive steps from the lowest (the most distant from the source) super-layer towards the highest super-layer with an index 0. During the bottom-up construction we process each cluster C such that we:

- provide a rank $rank(C)$ to the cluster,
- choose a node, called a *representative* of the cluster, in the highest layer of the cluster,
- select a unique *leading representative* in the lowest layer among representatives of cluster children of C, and
- define the *cluster path* as any shortest transmission path from the representative of C to the leading representative of its cluster children.

The tree of clusters is built on the basis of an arbitrary BFS tree formed of cluster nodes in consecutive super-layers. I.e., for each cluster, we choose a cluster in the neighboring higher super-layer, which will stand as its parent in the constructed tree of clusters. In what follows we show how each cluster in the k-th super-layer is processed in due course. Assume that all clusters in the $(k+1)$-th super-layer have been already processed and that we currently process the cluster $C_i^{(k)}$. Let $child(C_i^{(k)}) \subseteq \mathcal{C}^{(k+1)}$ be a (possibly empty) set of clusters in $\mathcal{C}^{(k+1)}$ whose parent is $C_i^{(k)}$. In the case when $child(C_i^{(k)}) = \emptyset$, the rank $rank(C_i^{(k)})$ of the cluster $C_i^{(k)}$ is set to 0 and an arbitrary node in the highest layer of the cluster is chosen as the representative of the cluster. In this case we do not define the cluster path and the leading representative due to the lack of cluster children. If, however, $child(C_i^{(k)}) \neq \emptyset$, i.e., the cluster $C_i^{(k)}$ is a parent cluster of one or more clusters at the $(k+1)$-th super-layer, we process the cluster as follows. Let $r_{max} = max\{rank(C) | C \in child(C_i^{(k)})\}$ be the maximal rank among all ranks of its cluster children. If at least two cluster children in $child(C_i^{(k)})$ have ranks with value r_{max}, i.e., $|\{C | C \in child(C_i^{(k)}) \wedge rank(C) = r_{max}\}| \geq 2$, the rank of $C_i^{(k)}$ is set to $r_{max} + 1$ and the leading representative is chosen arbitrarily among representatives of the cluster children. Otherwise, the rank is set to r_{max} and the representative of the cluster child with the rank r_{max} is chosen as the leading representative of the cluster. Let u be the chosen leading representative. Since each cluster is a union of pre-clusters, there is a node $v \in L_{k \cdot x} \cap C_i^{(k)}$ and a pre-cluster $S(v)$, s.t., $u \in S(v)$. We choose v as the representative of $C_i^{(k)}$ and one of the shortest transmission paths from v to u is set as the cluster path. Since $u \in S(v)$, the length of the cluster path is x. Note that certain clusters with the rank 0 can be formed of less than $x+1$ layers. In such shallow clusters cluster paths are not defined. Finally, the parent of the cluster is an arbitrary cluster of the $(k-1)$-th super-layer $\mathcal{C}^{(k-1)}$ that contains the representative v of the cluster $C_i^{(k)}$.

The following lemma follows directly from the construction of the cluster tree.

LEMMA 9. *The cluster tree has the following properties:*

1. *If $C_j^{(k-1)}$ is a parent cluster of $C_i^{(k)}$, it holds that*
$$rank(C_j^{(k-1)}) \geq rank(C_i^{(k)}),$$
2. *For any cluster $C_i^{(k)}$, it holds that*
$$|\{C | C \in child(C_i^{(k)}) \wedge rank(C) = rank(C_i^{(k)})\}| \leq 1,$$
3. *Let $C_i^{(k)}$ be a cluster and $C_j^{(k+1)} \in child(C_i^{(k)})$ be a cluster child of $C_i^{(k)}$, s.t., $rank(C_i^{(k)}) = rank(C_j^{(k+1)})$. Then, there is a cluster path of the length x in $C_i^{(k)}$ that connect the representative of $C_i^{(k)}$ with the representative $C_j^{(k+1)}$, which is the leading representative of $C_i^{(k)}$.*

We define a *reverse rank* of a cluster $C_i^{(k)}$ as $rank^*(C_i^{(k)}) = rank(C_0^{(0)}) - rank(C_i^{(k)})$. The following properties of (reverse) ranks in the tree of clusters follow from discussion in [8, 11].

LEMMA 10. *The greatest rank in the cluster tree is assigned to the root and its value is at most $\log n$. The greatest reverse rank of a cluster is also at most $\log n$. Each simple path from the root to any other cluster forms a non-increasing (non-decreasing) sequence of (reverse) ranks.*

4.3 Building the broadcasting schedule

The broadcasting schedule is implemented as a concurrent (interleaved) execution of two communication patterns of *fast transmissions* and *slow transmissions*. As we will explain later, it is important that fast transmissions do not interfere with slow transmissions and vice versa. Thus transmissions coming from different patterns are executed in disjoint time steps. A produced schedule consists of consecutive *composite rounds*, in which the first round is reserved for fast transmissions and the second one serves slow transmissions.

4.3.1 Pattern of fast transmissions

The main aim of fast transmissions is to disseminate the source message from the cluster representative to the leading representative in the same cluster along the cluster path. In order to avoid interference of simultaneous fast transmissions coming from clusters with different colors, the transmissions are scheduled according to the distance from the source and colors of clusters. In fact, a node on a cluster path has a licence to transmit only in very specific composite rounds. We may picture this permission to transmit as a (time) wave emitted by the source s and descending gradually along the consecutive BFS layers of the input IRG G. Let $t_e = d_I(G) + 2$ be a *period* of this wave, i.e., the frequency of issuing the permission. Further, consider a node $v \in L_z = L_{k \cdot x + y}$, for some $0 \leq y < x$. Let $\mathcal{C}_P(v) \subseteq \mathcal{C}^{(k)}$ be a collection of all clusters in $\mathcal{C}^{(k)}$, s.t., the node v belongs to their cluster paths. Note that we exclude here the nodes of the layer $L_{(k+1)\cdot x} = \mathcal{L}_k \cap \mathcal{L}_{k+1}$, i.e., the lowest layer of the k-th super-layer, from cluster paths of clusters in $\mathcal{C}^{(k)}$. Due to this exemption, fast transmissions of nodes are determined by a clustering in exactly one super-layer. The node v has a license to transmit during t-th composite round, as a part of the pattern of fast transmissions, if and only if $t \equiv (z + c \cdot t_e) \pmod{t_e \cdot \lceil \log n \rceil}$, where $c \in \{color(C) | C \in \mathcal{C}_P(v)\}$. I.e., c is a color of a cluster containing v on its cluster path.

Summarising, a node v contributes to fast transmissions in a composite round t if:

(1) v is already informed,

(2) v has an uninformed transmission neighbor,

(3) v has enough energy to transmit (e.g., in k-shot protocols v transmitted at most $k-1$ times so far),

(4) $t \equiv (z + c \cdot t_e) \pmod{t_e \cdot \lceil \log n \rceil}$, where $c \in \{color(C) | C \in \mathcal{C}_P(v)\}$ (v is allowed to transmit).

4.3.2 Critical composite rounds

The notion of a *critical composite round* refers the latest round when a node is prompted by the broadcast schedule to transmit during fast transmission pattern.

For a cluster $C_i^{(k)} \in \mathcal{C}^{(k)}$, we define a *critical composite round* as
$$t_{cc}(C_i^{(k)}) = color(C_i^{(k)}) \cdot t_e + 2 \cdot k \cdot t_e \cdot \lceil \log n \rceil + K(C_i^{(k)}),$$
where $K(C_i^{(k)})$ is the smallest integer, s.t.,

- $K(C_i^{(k)}) \geq 7 \cdot (d_I(G) + x) \cdot \log n \cdot t_b \cdot rank^*(C_i^{(k)})$, and
- $K(C_i^{(k)}) \equiv 0 \pmod{t_e \cdot \lceil \log n \rceil}$.

The parameter t_b corresponds to the length of one stage of slow transmissions and it will be established later. Concerning other parameters the following upper bounds can be established:

- $color(C_i^{(k)}) \cdot t_e$ is the maximum number of composite rounds during which the representative of a cluster has to wait before it starts passing the source message along its cluster path,
- $k \cdot t_e \cdot \lceil \log n \rceil$ is the maximum number of composite rounds wasted by cluster representatives in ancestor clusters while waiting for the first chance to transmit along their cluster paths,
- $rank^*(C_i^{(k)})$ is the maximum number of times the representatives of ancestor clusters were not chosen as the leading representative of their parent clusters, and
- $7 \cdot (d_I(G) + x) \cdot \log n$ is the maximum number of stages with slow transmissions (each stage being of length t_b) in which the cluster representative waits for the source message from its parent cluster in the case when its parent cluster has a different rank.

Finally, a *critical composite round* $t_c(v)$ for the node v ($v \in L_z$) is defined as follows:
$$t_c(v) = min\{z + t_{cc}(C_i^{(k)}) | C_i^{(k)} \in \mathcal{C}_P(v)\}.$$

4.3.3 Pattern of slow transmissions

Recall that the main purpose of slow transmissions is to disseminate the source message from informed cluster representatives and cluster paths to all other nodes in their clusters. Slow transmissions are performed in stages where each stage consists of a fixed number of composite rounds. During each stage, all nodes which are informed pass the source message on all their uninformed transmission neighbors. Hence, one stage of slow transmissions can be seen as information dissemination in a bipartite IRG.

Fix an algorithm $\mathcal{A}_b$ for information dissemination in bipartite IRGs. Let t_b be the maximal length of the schedule generated by $\mathcal{A}_b$ for arbitrary bipartite IRG which is a subgraph of the IRG G. The k-th stage of slow transmissions, for $k \geq 0$, starts in the composite round $k \cdot t_b$ and finishes in the composite round $(k+1) \cdot t_b - 1$, i.e., each stage lasts through exactly t_b composite rounds. In the first composite round of each stage we consider a bipartite IRG G_b, for which a schedule of transmissions is computed by the algorithm $\mathcal{A}_b$. The first part of G_b contains all informed nodes with uninformed transmission neighbors with the exception of all nodes whose critical composite rounds occur during this stage of slow transmissions. As we show later, if a node transmits in the pattern of fast transmissions, all its uninformed transmission neighbors become informed. However, this is not true for transmissions contributing to the pattern of slow transmissions. In particular, slow transmissions only guaranty that all uninformed transmission neighbors become informed at the end of the stage. But this could be too slow and inconsistent with the main aim of fast transmissions. The second part of the graph G_b is formed by all uninformed nodes connected by a transmission edge to a node in the first part.

4.4 Analysis of the broadcasting schedule

LEMMA 11. *Any fast transmission results in informing all transmission neighbors.*

PROOF. Fast and slow transmissions do not interfere due to adopted time multiplexing strategy.

Hence, the only case, when a transmission neighbor w of a transmitting node $u \in L_{z_u}$ does not receive a transmitted message from u in a composite round t, is the case when there is a node $v \in L_{z_v}$ transmitting simultaneously in the same round, s.t., an edge $(v, w) \in E_T \cup E_I$. Let c_u be the color of a cluster C_u which determines transmission of u in the composite round t. Note that the cluster C_u must contain u in its cluster path and u is not the last node on this path. Similarly, let c_v be the color of a cluster C_v which determines transmission of v in the composite round t. The scheduling mechanism for fast transmissions implies that

$$t \equiv (z_u + c_u \cdot t_e) \equiv (z_v + c_v \cdot t_e) \pmod{t_e \cdot \lceil \log n \rceil}.$$

If $c_u \neq c_v$, we get $|z_u - z_v| \geq t_e$ using the fact that $c_u, c_v \leq \lceil \log n \rceil$. Thus, $d_T(u, v) \geq t_e = d_I(G) + 2$. Since w is a transmission neighbor of u, the inequality $d_T(u, v) \geq d_I(G) + 2$ implies $d_T(w, v) \geq d_I(G) + 1$. Hence, there is no edge $(w, v) \in E_T \cup E_I$ that contradicts the assumption $(w, v) \in E_T \cup E_I$.

For $c_u = c_v$, the scheduling mechanism for fast transmissions implies $z_u \equiv z_v \pmod{t_e \cdot \lceil \log n \rceil}$. If $z_u \neq z_v$, we get $|z_u - z_v| \geq t_e \cdot \lceil \log n \rceil \geq t_e = d_I(G) + 2$. In this case we also get contradiction with the assumption $(w, v) \in E_T \cup E_I$. Hence, it remains to consider the case when $z_u = z_v$ and $c_u = c_v$. Recall that the fact that a node is a last node on a cluster path does not influence its activity in the pattern of fast transmissions. I.e., the activity of a node depends only on time and cluster colors in exactly one superlayer. Thus, there exists k such that $C_u, C_v \in \mathcal{C}^{(k)}$. Since $c_u = \mathit{color}(C_u) = \mathit{color}(C_v) = c_v$, the property (3) of the lemma 8 implies that $(v, w) \notin E_T \cup E_I$, which also contradicts the assumption. □

An important implication of Lemma 11 is the property that each node transmits in the pattern of fast transmissions at most once.

LEMMA 12. *Let S be a set of all informed nodes of the graph G at the beginning of a stage of slow transmissions. Each node that is a transmission neighbor of a node in S is informed by the end of this stage.*

PROOF. It is enough to only consider the nodes that have an uninformed transmission neighbor at the beginning of the stage of slow transmissions. Initially, all such nodes form an informed part of a bipartite IRG for this stage of slow transmissions with the exception of nodes whose critical composite rounds are scheduled during the stage of slow transmissions. Since every exceptional node has an uninformed transmission neighbor, Lemma 11 implies that it did not transmit during any of previous composite rounds. The node is informed. Thus, according to scheduling mechanism for fast transmissions, the node will transmit during its critical composite round at the latest. And Lemma 11 guaranties that its transmission neighbors will become informed. Finally, a schedule of slow transmissions constructed for this stage guaranties that transmission neighbors of all other nodes in the uninformed part become informed. □

LEMMA 13. *Assume that the algorithm $\mathcal{A}_b$ that generates schedules for stages of slow transmissions, produces k-shot schedules, i.e., where each node of informed part transmits at most k times. Then, a broadcasting schedule produced by the algorithm for IRG G is a k-shot broadcasting schedule.*

PROOF. We already know that every node acts as a transmitter in at most one stage of slow transmissions, where the schedule for that stage is produced by the algorithm $\mathcal{A}_b$. Thus if $\mathcal{A}_b$ produces a k-shot schedule, a node that acts during this stage as a transmitter transmits at most k times. In only problematic case when such a node is also due to contribute to fast transmission interleaved with slow transmissions of this stage, it simply never executes the schedule for slow transmissions. It awaits the round with the fast transmission and then informs all its transmission neighbors instantly. Since every node transmits in the pattern of fast transmissions at most once the limit of transmissions per node is also not exceeded in this case. □

LEMMA 14. *All transmission neighbors of a node v that belongs to a cluster path are informed by the end of its critical composite round $t_c(v)$.*

PROOF. The proof is done by induction on the layer number of the node.

First, we analyze the base case, i.e., the claim for a source $s \in L_0$. A reverse rank of the source's cluster is 0. If $t_b < t_c(s)$, the informed source participates in the first stage of slow transmissions. From the Lemma 12, all its transmission neighbors become informed by the end of the first stage. In the complementary case, when $t_b \geq t_c(s)$, the source does not participate in the first stage of slow transmissions. Since $t_c(v)$ is a composite round when the source has a permission to transmit, the source transmits and all its transmission neighbors get informed due to Lemma 11. Hence, in both cases, all transmission neighbors of the source are informed during the composite round $t_c(v)$ at the latest.

Now, we prove the claim for a node $v \in L_z$ under assumption that induction hypothesis holds, i.e., that the claim is true for every node $u \in L_y$, where $y < z$. We consider two cases: (1) when the node v is not a cluster representative, and the complementary case (2).

Lets analyze case (1) first. Let C be a cluster from $\mathcal{C}_P(v)$, s.t., $t_c(v) = z + t_{cc}(C)$ (see definition of the critical composite round). Recall that the node v lies on the cluster path in C. Since the node v is not a cluster representative, it has a predecessor $u \in L_{z-1}$ on the cluster path in C. Since $C \in \mathcal{C}_P(u)$ we also get $t_c(u) = min\{(z-1) + t_{cc}(C') | C' \in \mathcal{C}_P(u)\} \leq (z-1) + t_{cc}(C) < z + t_{cc}(C) = t_c(v)$. Thus, $t_c(u) < t_c(v)$. The induction hypothesis, for $u \in L_{z-1}$, implies that by the end of the composite round $t_c(u)$ all transmission neighbors of u are informed. The node v is a transmission neighbor of u. Therefore, the node v is informed by the end of composite round $t_c(u)$. If between composite rounds $t_c(u)$ and $t_c(v)$ there is room for at least one full stage of slow transmissions, it follows from Lemma 12 that all transmission neighbors of v are informed by the end of composite round $t_c(v)$. Otherwise, according to the scheduling mechanism for fast transmissions, the node v transmits in the composite round $t_c(v)$ at the latest. Hence, by lemma 11, all transmission neighbors of v are informed by the end of composite round $t_c(v)$.

It remains to analyze the case when the node $v \in L_z$ is a cluster representative. Since v is a cluster representative, it holds that $z = k \cdot x$ and $k \geq 1$. Let C be a cluster from $\mathcal{C}_P(v) \subseteq \mathcal{C}^{(k)}$, s.t., $t_c(v) = z + t_{cc}(C)$. Further, let $C' \in \mathcal{C}^{(k-1)}$ be the parent cluster of the cluster C, i.e., $C \in child(C')$. From property (1) of Lemma 8, it follows that $rank(C) \leq rank(C')$. Next, from the definition of reverse rank, we get $rank^*(C) \geq rank^*(C')$. Two subcases emerge: $rank^*(C) = rank^*(C')$ and $rank^*(C) > rank^*(C')$.

Assume first that $rank^*(C) = rank^*(C')$ and compare critical composite rounds t_{cc} of both clusters. Ranks of both clusters are the same, $C \in \mathcal{C}^{(k)}$, and $C' \in \mathcal{C}^{(k-1)}$. Therefore, we have holds $t_{cc}(C) - t_{cc}(C') = (color(C) - color(C')) \cdot t_e + 2 \cdot t_e \cdot \lceil \log n \rceil \geq t_e \cdot \lceil \log n \rceil$. The last inequality follows from the fact that any cluster color is a number smaller than $\lceil \log n \rceil$. Further, property (3) in Lemma 9 implies that v is the leading representative of the parent cluster C'. Hence, there is a predecessor u of the node v on the cluster path of the parent cluster C'. Since $C' \in \mathcal{C}_P(u) \subseteq \mathcal{C}^{(k-1)}$, it also holds that $t_c(u) \leq (z-1) + t_{cc}(C')$. Therefore, $t_c(v) - t_c(u) \geq (z + t_{cc}(C)) - ((z-1) + t_{cc}(C')) \geq 1 + (t_{cc}(C) - t_{cc}(C')) \geq 1 + t_e \cdot \lceil \log n \rceil$. Note, that from the induction hypothesis for u it follows that at the end of composite round $t_c(u)$ all transmission neighbors of the node u, including v, are informed. Since also $t_c(v) - t_c(u) \geq 1 + t_e \cdot \lceil \log n \rceil$, the node v has at least $1 + t_e \cdot \lceil \log n \rceil$ composite rounds to inform all its transmission neighbors after a composite round, when it becomes informed, is finished. If this time interval contains at least one full stage of slow transmissions, it follows from Lemma 12 that all transmission neighbors of v become informed by the end of the composite round $t_c(v)$. In the complementary case, we observe (see the scheduling mechanism for fast transmissions) that this time interval is longer than $t_e \cdot \lceil \log n \rceil$. It also contains a composite round when the node v is allowed to transmit in the pattern of fast transmissions according to the color of C. From Lemma 11, all transmission neighbors of the node v become informed.

Finally, assume $rank^*(C) > rank^*(C')$. Let $u \in L_{(k-1)\cdot x}$ be the representative of the parent cluster C'. By the induction hypothesis, all transmission neighbors of the node u are informed by the end of the composite round $t_c(u)$. Now, we estimate the length of a time interval between a composite round when all transmission neighbors of u are informed and a composite round when all transmission neighbors of v have to be informed. Since $rank^*(C) - 1 \geq rank^*(C')$, one can show that $t_c(v) - t_c(u) \geq 7 \cdot (d_I(G) + x) \cdot \log n \cdot t_b$. The node v is the representative of the cluster C. Therefore, it belongs to the parent cluster C' of cluster C, i.e., $v \in C'$. Also property (2) of the lemma 8 implies that $d_T(u, v) \leq 6 \cdot (d_I(G) + x) \cdot \log n$. Certainly, there exists a transmission neighbor u' of the node u such that $d_T(u', v) + 1 = d_T(u, v)$. For any transmission neighbor v' of the node v, it holds that $d_T(u', v') \leq d_T(u', v) + 1 = d_T(u, v) \leq 6 \cdot (d_I(G) + x) \cdot \log n$. Since the node u' is a transmission neighbor of the node u, it gets informed by the end of the round $t_c(u)$. Observe also, that the time interval between $t_c(u)$ and $t_c(v)$ contains at least $6 \cdot (d_I(G) + x) \cdot \log n$ complete stages of slow transmissions. And indeed, $t_c(v) - t_c(u) \geq 7 \cdot (d_I(G) + x) \cdot \log n \cdot t_b \geq (6 \cdot (d_I(G) + x) \cdot \log n) \cdot t_b + t_b$. Since $d_T(u', v') \leq 6 \cdot (d_I(G) + x) \cdot \log n$, Lemma 12 (iterated $6 \cdot (d_I(G) + x) \cdot \log n$ times) implies that the node v', a transmission neighbor of the node v, becomes informed in the composite round $t_c(v)$ at the latest. □

LEMMA 15. *Let G be a given undirected IRG and $s \in V(G)$ be a given source node. Let $\mathcal{A}_b$ be a polynomial-time algorithm which produces schedules of transmissions of length t_b, for information dissemination in bipartite IRGs that are subgraphs of G. There is a polynomial-time algorithm that generates schedules of transmissions with length*

$$4 \cdot ecc_T(s) + O(d_I(G) \cdot \log^3 n \cdot t_b).$$

Moreover, if $\mathcal{A}_b$ generates k-shot schedules in bipartite IRGs, then generated schedules for arbitrary IRGs are also k-shot schedules.

PROOF. Lemma 14 implies that each cluster representative v is informed in the composite round $t_c(v)$ at the latest. Since ranks of clusters are at most $\log n$ and the number of super-layers is $\lceil \frac{ecc_T(s)+1}{x} \rceil$, it follows that $max\{t_c(v) | v \in V(G)\} \leq ecc_T(s) + 3 \cdot \lceil \frac{ecc_T(s)+1}{x} \rceil \cdot t_e \cdot \lceil \log n \rceil + O((d_I(G) + x) \cdot \log^2 n \cdot t_b)$. Thus, all cluster representatives are informed during this composite round at the latest. From property (2) in Lemma 8 we can conclude that the transmission distance between a node and the nearest cluster representative is at most $6 \cdot (d_I(G) + x) \cdot \log n$. Thus, by Lemma 12, all nodes in the network become informed after at most $6 \cdot (d_I(G) + x) \cdot \log n$ subsequent stages of slow transmissions. These stages take in total at most $O((d_I(G) + x) \cdot \log n \cdot t_b)$ composite rounds. Therefore, all nodes in the network can be informed in at most $ecc_T(s) + 3 \cdot \lceil \frac{ecc_T(s)+1}{x} \rceil \cdot t_e \cdot \lceil \log n \rceil + O((d_I(G) + x) \cdot \log^2 n \cdot t_b)$ composite rounds. Now choosing $x = 3 \cdot t_e \cdot \lceil \log n \rceil$ we get $3 \cdot \lceil \frac{ecc_T(s)+1}{x} \rceil \cdot t_e \cdot \lceil \log n \rceil = ecc_T(s) + O(t_e \cdot \log n)$.

Note also that $t_e = d_I(G)+2$. Hence, the total length of the schedule is $ecc_T(s)+(ecc_T(s)+O(d_I(G)\cdot\log n))+O((d_I(G)+(d_I(G)+2)\cdot\log n)\cdot\log^2 n\cdot t_b) = 2\cdot ecc_T(s)+O(d_I(G)\cdot\log^3 n\cdot t_b)$. Finally, since each composite round consists of two rounds, the thesis of the lemma is proved.

Preprocessing phase (construction of pre-cluster graphs, clustering, cluster tree, cluster paths, etc.) is realized in time $O(n^4)$ utilising Floyd-Warshall algorithm, BFS traversals and the clustering algorithm from [8]. The broadcasting schedule is generated by round-by-round simulation of communication patterns (fast and slow transmission). I.e., when we simulate a round, we compute a set of transmitting informed nodes with respect to defined rules and then we determine new set of informed nodes due to known network topology. The simulation time of one round is bounded from above by $O(n^2)+T(\mathcal{A}_b)$, we simulate at most $O(n\cdot\log^3 n\cdot t_b)$ rounds, and $t_b = O(n)$. It follows that the time complexity of the algorithm is polynomial. $\square$

The main result of this paper follows from Lemma 15 and Theorem 3.

THEOREM 16. *Let G be a given undirected IRG G where $s \in V(G)$ is the source node. There is a polynomial-time algorithm generating 1-shot schedules of transmissions that accomplishes broadcasting task in time*

$$4\cdot ecc_T(s)+O(\Delta\cdot d_I(G)\cdot\log^4 n) = 4\cdot D_T+O(\Delta\cdot d_I(G)\cdot\log^4 n).$$

5. CONCLUSION

We studied broadcasting in known topology radio networks modelled by undirected graphs, where the interference range of a node is likely to exceed its transmission range. The focus was on the design of fast broadcasting schedules that are also energy efficient, i.e., based on limited number of transmissions at each node. The main result of the paper is a polynomial-time algorithm that computes a 1-shot broadcasting schedule of length at most $4 \cdot D_T + O(\Delta \cdot d_I \cdot \log^4 n)$ for networks with arbitrary topology.

Several interesting problems motivated by our work remain open. This includes efficient communication algorithms in a model in which the sub-network induced by transmission edges is known, however, the location of interference edges is unspecified. Also a model in which the entire topology of IRG is unknown is worth investigation. One could also consider several probabilistic models, where, e.g., the topology of connections is represented by a random graph or the interference edges reoccur with a certain probability. Finally, another interesting avenue of research explorations is a study on broadcasting in known radio networks with long-range interference represented as directed graphs.

6. REFERENCES

[1] N. Alon, A. Bar-Noy, N. Linial, and D. Peleg. A lower bound for radio broadcasting. *Journal of Computer and System Sciences 43*, 1991, pp. 290-298.

[2] P. Berenbrink, C. Cooper, and Z. Hu. Energy Efficient Randomised Communication in Unknown AdHoc Networks. In *Proc. 19th ACM Symp. on Parallelism in Algorithms and Architectures*, 2007, pp. 250-259.

[3] J.-C. Bermond, J. Galtier, R. Klasing, N. Morales, and S. Perennes. Hardness and approximation of gathering in static radio networks. *Parallel Processing Letters* 16, 2006, pp. 165-184.

[4] I. Chlamtac and S. Kutten. On broadcasting in radio networks - problem analysis and protocol design. *IEEE Transactions on Communications* 33, 1985, pp. 1240–1246.

[5] I. Chlamtac and S. Kutten. The wave expansion approach to broadcasting in multihop radio networks. *IEEE Transactions on Communications* 39, 1991, pp. 426-433.

[6] T. Cormen, Ch. Leiserson, R. Rivest, and C. Stein. Introduction to Algorithms. Second Edition. MIT Press and McGraw-Hill, 2001. ISBN 0-262-03293-7. Section 8.2: Counting sort, pp. 168-170.

[7] M. Elkin and G. Kortsarz. Improved schedule for radio broadcast. In *Proc. 16th ACM-SIAM Symposium on discrete Algorithms* (SODA'2005), 2005, pp. 222-231.

[8] I. Gaber and Y. Mansour. Centralized broadcast in multihop radio networks. *Journal of Algorithms* 46 (1), 2003, pp. 1-20.

[9] F. Galčík. Centralized communication in radio networks with strong interference. In *Proc. SIROCCO* 2008, LNCS 5058, 2008, pp. 277-290.

[10] L. Gąsieniec, E. Kantor, D. Kowalski, D. Peleg, and Ch. Su. Time efficient k-shot broadcasting in known topology radio networks. *Distributed Computing* 21 (2008), pp. 117-127.

[11] L. Gąsieniec, D. Peleg, and Q. Xin. Faster communication in known topology radio networks. In *Proc. 24th Annual ACM Symp. on Principles of Distributed Computing* (PODC'2005), 2005, pp. 129-137.

[12] O. Goussevskaia, T. Moscibroda, and R. Wattenhofer. Local broadcasting in the physical interference model. In *Proc. DIALM-POMC'08*, 2008, pp. 35-44.

[13] P. Gupta and P.R. Kumar. The Capacity of Wireless Networks. *IEEE Transactions on Information Theory* 46(2), 2000, pp. 388-404.

[14] D. Kowalski and A. Pelc. Centralized deterministic broadcasting in undirected multi-hop radio networks. In *Proc. APPROX*, LNCS 3122, 2004, pp. 171-182.

[15] D. Kowalski and A. Pelc. Optimal deterministic broadcasting in known topology radio networks. *Distributed Computing* 19, 2007, pp. 185-195.

[16] T. Muetze, P. Studedi, F. Kuhn, and G. Alonso. Understanding Radio Irregularity in Wireless Networks. In *Proc. SECON'08*, 2008, pp. 82-90.

[17] M. Onus, A. Richa, K. Kothapalli, and Ch. Scheideler. Efficient Broadcasting and Gathering in Wireless Ad-Hoc Networks. In *Proc. ISPAN 2005*, 2005, pp. 346-351.

[18] D. Peleg. Time-Efficient Broadcasting in Radio Networks: A Review. In *Proc. ICDCIT 2007*, LNCS 4882, 2007, pp. 1-18.

[19] A. Pelc. Broadcasting in radio networks. *Handbook of Wireless Networks and Mobile Computing*, I. Stojmenovic, Ed. John Wiley and Sons, Inc., New York, 2002, pp. 509-528.

[20] D. Welsh and M. Powell. An upper bound for the chromatic number of a graph and its application to timetabling problems. *The Computer Journal* 10 (1), 1967, pp. 85-86.