On radio communication in grid networks

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Abstract. *The paper considers deterministic communication in radio networks whose underlying reachability graphs are undirected 2-dimensional grids. Radio communication is synchronous and differs from other communication models in a way, how simultaneously incoming messages are processed. A node receives a transmitted message iff exactly one its neighbor transmits in a given time slot. In our setting, we assume that the nodes (networks stations) are spread in a region in a regular way. Particularly, they are located at grid points of a square mesh. Initial knowledge of a node is limited only to its unique identifier. The node is not aware of its position in the grid as it was assumed in other papers. We design an algorithm completing the broadcasting task in $O(ecc + \log N)$ rounds using total $O(n + \log N)$ transmissions, where ecc is the eccentricity of the source and N is an upper bound of unique integer identifiers assigned to the nodes. Moreover, we present a modification of the algorithm that solves a task of computation grid coordinates of each node in the asymptotically same time. All presented algorithms are asymptotically optimal according to both considered complexity measures.*

1 Introduction

A *radio network* is a collection of autonomous stations that are referred as *nodes*. The nodes communicate via sending messages. Each node is able to receive and transmit messages, but it can transmit messages only to nodes, which are located within its transmission range. The network can be modelled by a directed graph called *reachability graph* $G = (V, E)$. The vertex set of G consists of the nodes of the network and two vertices $u, v \in V$ are connected by an edge $e = (u, v)$ iff the transmission of the node u can reach the node v. In such a case the node u is called a *neighbor* of the node v. If the transmission power of all nodes is the same, then the reachability graph is symmetric, i.e. a symmetric radio network can be modelled by an undirected graph. In what follows, we focus on communication in a radio grid, i.e. an underlying reachability graph of the radio network is a grid graph. Our aim is to model radio communication in the real-world case when the nodes (networks stations) are spread in a region in a regular way. Particularly, they are located at all grid points of a square mesh and the transmission radius of a node is equal to 1.

Definition 1. *For $p, q > 1$, an undirected graph $G_{p,q}$ is called a grid graph iff:*

$$V(G_{p,q}) = \{(i,j) | 0 \le i < p, 0 \le j < q\}$$
$$E(G_{p,q}) = \{((i,j),(i',j')) | (i' = i \wedge j' = j \pm 1) \vee (i' = i \pm 1 \wedge j' = j)\}.$$

Radio communication is synchronous and differs from other communication models in a way, how simultaneously incoming messages are processed. A node receives a transmitted message iff exactly one its neighbor transmits in a given time slot. In particular, the nodes of a network work in synchronized steps (time slots) called *rounds*. In every round, a node can act either as a *receiver* or as a *transmitter*. A node u acting as transmitter sends a message, which can be potentially received by each its neighbor. In the given round, a node, acting as a receiver, receives a message only if it has exactly one transmitting neighbor. The received message is the same as the message transmitted by the transmitting neighbor. If in the given round, a node u has at least two transmitting neighbors we say that a *collision* occurs at node u. We assume that the nodes cannot distinguish an *interference noise* (at least two transmitting neighbors) and a *background noise* (no transmitting neighbor), i.e. the collision at the node u seems for the node u as the round in which no its neighbor transmits. We discuss communication complexity of tasks. Hence the time for determining an action in a round is insignificant, i.e. the complexity of an internal computation is not considered.

The goal of *broadcasting* is to distribute a message from one distinguished node, called a *source*, to all other nodes. Remote nodes of the network are informed via intermediate nodes. The time, required to complete an operation, is important and widely studied parameter of mostly every communication task. Since the nodes (radio stations) used in real-world applications are very often powered by batteries, another important complexity measure is a *energetic complexity*. It is denoted as the total number of transmissions that occur during the execution of an algorithm in the network.

* Author thanks to an anonymous reviewer for his suggestions and references concerning 2-distance coloring.

In this paper, we consider both complexity measures: time and energetic complexity. The complexity of an algorithm is described as a function of three parameters of radio network: the number of nodes (denoted as n), the unknown upper bound of identifiers assigned to the nodes (denoted as N), and the largest distance from the source to any other node of the network (denoted as *ecc*).

According to different features of stations forming a radio network, many models of radio networks have been developed and discussed. They differ in used communications scenarios and in an initial knowledge assumed for nodes. The overview of the models of radio networks can be found e.g. in [9]. Through this paper, we focus on the deterministic distributed algorithms in unknown radio networks with a grid topology. An initial knowledge of a node is limited only to its unique integer identifier (label). The node is not aware of its position in the grid. Moreover, neither the size of an underlying reachability graph (the number of nodes) nor an upper bound of identifiers are known to the nodes. Most of previous works deal with the assumption that $N = O(n^p)$, for a constant $p > 0$. In this paper, we consider that there is no relationship between N (an upper bound of identifiers) and n (the number of nodes), except trivial $N \geq n$. It models the case when each network station possesses a unique factory identifier (e.g. MAC address) of large scale, but the number of nodes forming a network is relatively small. Note that considered settings are not a typical real-world case since the network topology is fixed to a specific graph topology. On the other hand, all considered assumptions cover the case immediately after appropriate arrangement of nodes (with only a factory initial knowledge) in an area. In this sense, the algorithms proposed in this paper could be seen as the algorithms computing an auxiliary information that serves also for establishment of a fast communication mechanism.

1.1 Related Work

The first distributed algorithms for unknown radio networks were presented by Diks et al. in [4]. The authors considered a restricted class of networks having the nodes situated in a line. Moreover, they assumed that each node could reach directly all nodes within a certain distance. Systematic study of deterministic distributed broadcasting has been initiated by Chlebus et al. in [2]. Recently, Czumaj and Rytter [3] gave currently best deterministic broadcasting algorithm that completes the task in time $O(n \log^2 D)$ where D is the diameter of the reachability graph. Note that $D \leq 2.ecc$. In the case when underlying reachability graph is symmetric, i.e. radio networks is modelled by a undirected graph, more efficient broadcasting algorithm has been constructed. In [7], Kowalski and Pelc gave a $\Omega(n \frac{\log n}{\log (n/D)})$ lower bound for symmetric radio networks and designed a $O(n \log n)$-rounds broadcasting algorithm.

Radio communication in networks with grid topology was investigated by Kranakis et al. in [8]. The authors discussed fault tolerant broadcasting in known topology radio grid networks (each node knows its coordinates and dimensions of the grid graph). Bermond et al. [1] considered modified model of communication in know topology radio grid networks that follows from a problem proposed by France Telecom. In this model, transmission of a node causes an interference in some nodes that are not within the transmission range of the node. This region is called an *interference range* of the node. In standard model, the interference range and the transmission range of a node are the same. The authors studied a *gathering task* where the goal is to gather information from all nodes of the network to a distinguished central node.

2 Radio Broadcasting in Grid Networks

In this section, we focus on the broadcasting task in unknown radio networks whose underlying reachability graph is a grid graph $G_{p,q}$, for $p, q > 1$. Considering this setting, we design a deterministic distributed broadcasting algorithm that completes the task asymptotically optimal in $O(ecc + \log N)$ rounds. The algorithm consists of three parts. At first, the source selects one of its neighbors during the first part. The goal of the second part is to compute an initialization information which is later used in third part of the broadcasting algorithm. We shall compute the initialization information only in the neighbors of the source and in the nodes that have in their neighborhood two neighbors of the source. Finally, the third part of the algorithm disseminates a source message through the network. Simultaneously, a control information that is similar to the initialization information, is computed for newly informed nodes.

Definition 2. *A node is referred to as a 2-neighbor of the source iff it is adjacent with two neighbors of the source.*

2.1 Common Subroutines and Techniques

In this subsection, we present some supplementary techniques that are applied in the first and the second part of the broadcasting algorithm.

Definition 3. *Let v be a node with identifier $ID(v) > 0$ and let $(a_1, a_2, \ldots, a_k)_2$ be a binary representation of the number $ID(v)$. An infinite binary sequence $1, a_k, a_{k-1}, \ldots, a_2, a_1, 0, 0, \ldots$ is called a transmission sequence corresponding to the identifier $ID(v)$.*

Example 1. The transmission sequence corresponding to the identifier $11 = (1011)_2$ is $1, 1, 1, 0, 1, 0, 0, 0, \ldots$.

Note that the previous definition implies that transmission sequences corresponding to different identifiers differ at least in one position.

Consider the following case. A node u has at least one and at most two active (participating) neighbors, say v and w, that become informed in some unknown rounds (possibly different). We have to deliver an information from one of participating neighbors to the node u as soon as possible. This task can be easily solved applying previously defined transmission sequences. Suppose that a participating neighbor becomes informed in the round 0. In the i-th round, it transmits its information iff the i-th element of the transmission sequence corresponding to its identifier is equal to 1. This subroutine is referred to as *TAI* (transmission according to identifier). We show that at most $O(\log Max(ID(v), ID(w)))$ rounds are enough to inform the node u by one of its participating neighbors.

Lemma 1. *The TAI-subroutine completes the task in at most $O(\log Max(ID(v), ID(w)))$ rounds. The total number of transmissions is $O(\log Max(ID(v), ID(w)))$.*

Proof. Suppose that the node v is informed in the round i and the second participating neighbor w (if exists) is not already informed in this round. Since the first element of the transmission sequence is always 1, v transmits in the round $i + 1$ due to the *TAI*-subroutine. The node w does not transmit in the round $i + 1$ and thus u becomes informed. Now suppose that v and w are informed simultaneously in the round i. According to *TAI*, they transmit in the round $i+1$, but the interference causes that u is not informed. Since $ID(v) \neq ID(w)$, the transmission sequences corresponding to $ID(v)$ and $ID(w)$ are different. Hence, there must be an index j such that j-th elements of their binary transmission sequences are different. It implies that exactly one participating neighbor of u transmits in the round corresponding to the index j (i.e. in the round $i + j$). Therefore the node u is informed in this round. □

In order to avoid interaction during a simultaneous execution of several communication tasks, we can use a time division multiplexing strategy. Particularly, the i-th task from the set of k tasks is executed only in each round j such that $j \equiv i \,(mod\, k)$. In our setting, we do not allow spontaneous transmission, i.e. a node (except the source or the initiator) cannot transmit before successful receiving of a message transmitted by other node. If we include the number of actual round modulo k in each transmitted message, newly informed nodes can synchronize. Thus all nodes participating in the algorithm can simultaneously execute the same task in a given round.

2.2 Selection of a Neighbor of the Source

Now, we describe an algorithm that selects one of the neighbors of the source. We shall utilize the *TAI*-subroutine. If a message which is transmitted in the *TAI*-subroutine, includes an identifier of sender, successful receiving of the message implies that the receiver can select one of the senders. *TAI*-subroutine works only in the case when there are at most two participating senders. On the other hand, the source has at most 4 neighbors in the grid graph. Hence, a direct application of the *TAI*-subroutine is not possible. With the assistance of 2-neighbors of the source, we split the selection process into several applications of *TAI*-subroutine, where at most 2 nodes participate in process of transmitting information towards a specific node.

In the first round, the source transmits an initialization message. It is received exactly by all neighbors of the source. Subsequently, we start 4 simultaneous tasks (each of them in a separate time slot modulo 4). In the first task, each neighbor of the source tries to inform adjacent 2-neighbors by its identity. Since exactly two neighbors of the source have to transmit towards a 2-neighbor, we utilize the *TAI*-subroutine. After at most $O(\log N)$ rounds, at least one 2-neighbor becomes informed. Note that the nodes are not aware of the fact whether they are 2-neighbors of the source or not. We can solve this problem by a modification of the *TAI*-subroutine in such a way that the first transmission of a node contains a special message. If a node receives this special message, it knows that it is not 2-neighbor. Indeed, all neighbors of the source transmit in the second round of the algorithm (the first round of the *TAI*-subroutine) the special message that due to interference cannot be received by 2-neighbors. An informed 2-neighbor starts its activity during the second task and ignores all messages received in the rounds assigned for the first task. Particularly, it attempts to send a message that includes the received identifier of one its neighbor. Again, we utilize the *TAI*-subroutine. All nodes, except neighbors of the source, ignore transmitted messages during execution of this task. Further, a neighbor of the source that receives its identifier in a round of the second task, starts an execution of the third task and attempts to send its identifier in a message to the source utilizing the *TAI*-subroutine. If a neighbor of the source has received identifier of other node, it finishes its participation in all tasks, expect the fourth task. Moreover, each node that is active in the third task transmits a message in all rounds of the first task. Indeed, it blocks receipt of a message by its adjacent 2-neighbors. The source acknowledges the receipt of a identifier of one its neighbor by transmission of a message in the round reserved for the fourth task. This transmission stops execution of the first and the third task by all neighbors of the source because the selection task is accomplished. Besides, the neighbors ignore all received messages transmitted in rounds of the second task. The goals of tasks can be summarized as follows:

- first task - neighbors of the source send their identifiers towards 2-neighbors
- second task - 2-neighbors of the source send received identifier (that was received in the execution of the first task) towards neighbors of the source
- third task - neighbors of the source that are informed in a round of the second task, send their identifiers towards the source
- fourth task - the source acknowledges selection of a neighbor

Lemma 2. *The source selects one of its neighbors in* $O(\log N)$ *rounds and using total* $O(\log N)$ *transmissions, where* N *is the unknown upper bound of identifiers assigned to the nodes of the network.*

Proof. Time and energetic complexity of this part of the algorithm are obvious. The proof of correctness will appear in the full version of the paper [6]. It is based on the case analysis of all possible states (and future transmissions due to the algorithm) in the round when a 2-neighbor of the source has received a message at the first time. □

2.3 Computation of Initialization Information

During this part, an initialization process is performed. The goal is to mark neighbors of the source by distinct labels from the set $\{A, B, C, D\}$ in such a way that a node marked by A (B) has not a common neighbor with the node marked by C (D, respectively). Furthermore, we require that each 2-neighbor of the source knows the labels of both adjacent neighbors of the source. Thus these nodes (2-neighbors of the source) can be marked by distinct labels from the set $\{AB, BC, CD, AD\}$. Desired initial labelling of the nodes is described on figure 1.

Note that only neighbors and 2-neighbors of the source participate in this part of the algorithm. All other nodes ignore transmitted messages.

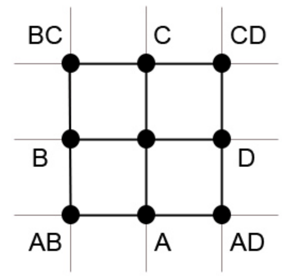

Fig. 1. The initial marking of nodes

1. The source transmits an initialization message containing the identifier of a selected neighbor.
2. Selected neighbor transmits a message and marks itself with label A.
3. 2-neighbors of the source that received the message in the previous round, retransmit the received message. A neighbor of the source that does not receive a message in this round, marks itself with the label C.
4. Each unmarked neighbor of the source executes the *TAI*-subroutine. It transmits messages containing its identifier as an information content of the node (in the sense of *TAI*). Execution of the *TAI*-subroutine is interleaved with rounds that are reserved for the source. In one of this rounds, the source informs the nodes participating on *TAI* that an unmarked neighbor is selected. This notification is realized by a transmission of the identifier of a selected (unmarked) neighbor.
5. Selected unmarked neighbor sets its label to B and the unselected neighbor (if exists) to D.
6. In one of 4 rounds, each neighbor transmits its label in a round that is designated for its labels.
7. 2-neighbors of the source set the labels according to labels received in previous rounds.

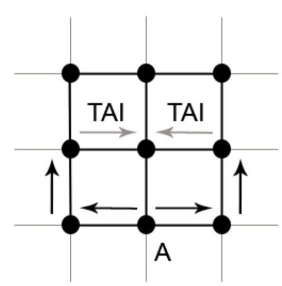

Fig. 2. Scheme of the initial marking computation

It is easy to see that the initialization schema works properly even the source has less than 4 neighbors. All steps, except step 4, require constant number of rounds. The step 4 is accomplished in $O(\log N)$ rounds due to lemma 1. We summarize the time complexity of this part that computes initialization information in the following lemma.

Lemma 3. *The initialization information can be computed in $O(\log N)$ rounds with the energetic complexity $O(\log N)$ transmissions.*

2.4 Dissemination of the Source Message

This part of broadcasting algorithm works in phases. The goal of each phase is to disseminate a source message to the nodes that are in neighborhood of nodes informed during the previous phase. Furthermore, newly informed nodes compute an auxiliary information that is used to arrange their transmissions in the next phase. The auxiliary information has a similar structure as the initialization information. Particularly, the auxiliary information is a label of a node. Each node can be marked by a label from the set $\{A, B, C, D, A^+, B^+, C^+, D^+, AB, BC, CD, AD\}$. The source message is disseminated from the source in a wave that has a shape of square. All nodes located on one side of square have the same label, however each side is marked by different label. The corner nodes are marked by a label from the set $\{AB, BC, CD, AD\}$. Intuitively, a label of a corner node expresses that the node belongs to two sides (directions of the broadcasting wave). We mark the nodes that are adjacent with corner nodes, by a label with + sign.

Initially, we change labels of nodes in the neighborhood of the source in such a way, that the label A is changed to A^+, B to B^+, C to C^+, and D to D^+. This transformation guarantees that the labels of nodes are compatible with the semantic description of labels stated above. In the first round of this part, neighbors and 2-neighbors of the source are considered as the nodes that were informed in the previous phase.

We assume that each transmitted message contains the source message and the number of actual round of executed phase. Each phase takes 5 communication rounds:

1. A node informed in the previous phase and marked by the label A, A^+, AB, AD, C, C^+, BC, or CD transmits a message containing its label.
2. A node informed in the previous phase and marked by the label B, B^+, C, or C^+ transmits a message containing its label.
3. The nodes that receive a label in the round 1 or 2 of the phase and are not yet informed, set the label to the received label. These nodes are referred to as newly informed nodes of given phase. Each newly informed node that received the label A^+, B^+, C^+, or D^+ in the round 1 or 2 of the given phase, transmits its label.
4. Newly informed nodes marked by label AB, AD, BC, or CD that received in the previous round the label A^+ or C^+ transmits their labels (received in round 1 or 2).
5. Newly informed nodes marked by label AB, AD, BC, or CD that received in the previous round the label B^+ or D^+ transmits their labels (received in round 1 or 2).

At the end of the phase, we change labels of some newly informed nodes. No communication is required in this step. First, the nodes with label A^+, B^+, C^+, or D^+ change the label to A, B, C, or D, respectively. Next, the nodes with label AB, AD, BC, or CD change the label to a label received in the round 3. Finally, if a node has received messages transmitted in the rounds 4 and 5, it sets its label to the received value and is considered as a newly informed node of the given phase. Note that the label received in the round 4 and in the round 5 is the same.

It is easy to see that the number of phases is limited by the eccentricity ecc of the source. Each phase takes constant number of round and thus the third part of algorithm is completed in $O(ecc)$ rounds. Each node transmits constantly many times. It implies that energetic complexity of this part is $O(n)$ transmissions.

2.5 Complexity of the Broadcasting Algorithm

The time and energetic complexity of designed broadcasting algorithm is summarized in the following theorem.

Theorem 1. *Consider a radio network such that its underlying reachability graph is a grid graph. There is a distributed deterministic algorithm that completes the broadcasting task in $O(ecc+\log N)$ rounds using total $O(n+\log N)$ transmissions, where n is the number of nodes and N is the unknown upper bound of identifiers in the network. Moreover, designed algorithm is asymptotically optimal.*

Proof. The first part of algorithm (selection of a neighbor of the source) takes $O(\log N)$ rounds and uses $O(\log N)$ transmissions. Time complexity of the second part (computation of the initial information) is $O(\log N)$ rounds. Energetic complexity of this part is $O(\log N)$ transmissions. Finally, the third part of algorithm that disseminates the source message, takes $O(ecc)$ rounds and uses $O(n)$ transmission. Therefore, the time complexity of the broadcasting algorithm is

$O(ecc+\log N)$ rounds. Summing total number of transmissions in each part of algorithm we obtain that at most $O(n+\log N)$ messages are transmitted by nodes during the execution of the algorithm.

Note that it is usually assumed that $N = O(n^p)$, for a constant $p > 0$. In grid graphs, it holds that $ecc \geq \sqrt{n} \geq \log n$ for sufficiently large n. Hence $O(\log N) = O(\log n) = O(ecc)$. In our setting, parameter N cannot be bounded in this way. Let fix a deterministic broadcasting algorithm. One can show that there is such an assignment of identifiers to the nodes of a grid radio network $G_{3,3}$ that the broadcasting task cannot be completed in less that $\Omega(\log N)$ rounds. The proof could be obtained by adaptation of the argument that was used in the proof of $\Omega(n\frac{\log n}{\log (n/D)})$ lower bound of the time complexity for the broadcasting task in [7]. Complete proof will appear in the full version of the paper [6]. □

2.6 Algorithm for Acknowledged Broadcasting

Note that the broadcasting algorithm presented in the previous section is not acknowledged, i.e. the source is not aware of the round when the broadcasting task is completed. Furthermore, the source is not able to compute the duration of the algorithm, because parameters of the radio network are unknown for the nodes. Presented principles of the constructed broadcasting algorithm allow to extend the algorithm to an algorithm completing the acknowledged broadcasting task. We present modified algorithm only briefly. The modification is based on the following. First, we add new labels A^*, B^*, C^*, and D^* to the set of labels that is used to mark the nodes. In each phase, new labels are used to mark one node, called a *progress node*, in each direction of the broadcasting wave. In particular, the progress nodes will form a cross with the center in the source. The nodes forming a limb of a cross are marked by the same label and each limb of a cross is marked by a different label. We append a new round to each phase of algorithm. In this round, each active progress node informs its neighboring progress node which is closer to the source, about the fact that the broadcasting task in the given direction is not yet accomplished. If an active progress node does not receive a message in this round during an appropriate phase, it becomes inactive. Since the nodes located on the border of grid do not inform new nodes, a progress node on this border does not receive a message in this round. It causes a chain of continuous deactivations of progress nodes. Finally, there must be a phase in which the source is notified (by silent) about completing broadcasting in the given direction. If the source is informed that disseminations of the source message are completed in all direction, it knows that all nodes are informed. In order to avoid interference during the last round, we have to schedule transmissions in appropriate manner. Let v to be a progress node that has been informed in the i-th phase (the number of the current phase must be included in each transmitted message). We define a number $P(v)$ as follows:

- $P(v) = (i \; mod \; 3) \; mod \; 4$, if v is marked by A^*
- $P(v) = (i \; mod \; 3+1) \; mod \; 4$, if v is marked by B^*
- $P(v) = (i \; mod \; 3+2) \; mod \; 4$, if v is marked by C^*
- $P(v) = (i \; mod \; 3+3) \; mod \; 4$, if v is marked by D^*

An active progress node v transmits a message in the last round of the j-th phase iff $j \equiv P(v) \; (mod \, 4)$.

It is easy to see that asymptotical time and energetic complexity are preserved by this modification.

3 Computation of Grid Coordinates

Since we consider radio networks with a regular topology, it could be assumed that the radio network is static. It means that the topology of the network remains unchanged for a long time period. This assumption heads towards the issue of computation of an communication structure for the collision-free communication. As we show later, the grid coordinates of nodes can serve as the basic information for a collision-free communication schema. In this section we present a distributed algorithm which computes grid coordinates of each node. The algorithm is a modification of the previously presented broadcasting algorithm. Particularly, it takes advantage of the auxiliary information computed during the third part of the broadcasting algorithm. We assume that the task of computation of grid coordinates is initiated by a distinguished node, called a *initiator*.

The algorithm consists of 3 parts. First two parts of the algorithm are identical to first two parts of the broadcasting algorithm. After this two part, an initialization information is computed. Now, we assign to the source coordinates $[0,0]$ and to its neighbor marked by A^+ (B^+, C^+, D^+) coordinates $[0,1]$ ($[-1,0]$, $[0,-1]$, $[1,0]$, respectively). It is easy to see that the assignment of coordinates is correct. Indeed, it follows from the way how the nodes are marked by labels during the second part of the algorithm. Similarly, we assign to a node marked by AB (BC, CD, AD) coordinates $[-1,1]$ ($[-1,-1]$, $[1,-1]$, $[1,1]$, respectively). Notice that in the broadcasting algorithm the labels of nodes store an information about direction in which the source message is disseminated. We shall use the sense of direction in order to compute grid coordinates of nodes according to information received from some their neighbors.

We modify content of messages sent in the third part of the broadcasting algorithm in the following way. Each message that is transmitted in the round 1 or 2 of the phase, contains coordinates of the sender. During execution of the third part of the algorithm, we preserve an invariant that each informed node (in sense of broadcasting algorithm) has already computed its grid coordinates. Thus the nodes transmitting in first two rounds of a phase have already computed their coordinates. Moreover, we modify messages sent in the round 4 and 5 of a phase. We attach grid coordinates received in one of first two rounds of the phase (in fact it happens in exactly one of those rounds) to the transmitted message. Finally at the end of the phase, the newly informed nodes compute their coordinates. Let $[x, y]$ be the coordinates which were included in a message received in the round 1, 2, 4, or 5 of a phase. We apply the following rules to set the coordinates of a newly informed node:

– label of node A or A^+: $[x, y + 1]$
– label of node B or B^+: $[x - 1, y]$
– label of node C or C^+: $[x, y - 1]$
– label of node D or D^+: $[x + 1, y]$
– label of node AB: $[x - 1, y + 1]$
– label of node BC: $[x - 1, y + 1]$
– label of node CD: $[x + 1, y - 1]$
– label of node AD: $[x + 1, y + 1]$.

Theorem 2. *Consider an unknown radio network such that its underlying reachability graph is a grid graph. There is a distributed deterministic algorithm that computes grid coordinates of each node in $O(ecc + \log N)$ rounds with total $O(n + \log N)$ transmissions, where n is the number of nodes and N is the unknown upper bound of identifiers in the network. Designed algorithm is asymptotically optimal.*

Proof. Correctness of the algorithm follows from the properties of the broadcasting algorithm and the rules for computation of coordinates of newly informed nodes. Since we do not allow spontaneous transmission (i.e. to participate on algorithm before receiving a message from other node), the task cannot be accomplished in better time (and energetic complexity) than the broadcasting task. It implies asymptotical optimality of designed algorithm. □

Note that we can design an algorithm in which the initiator of the computation is notified that the task is completed. It could be achieved by a similar modification of acknowledged broadcasting algorithm for radio networks with grid topology that is presented in the section 2.6

4 Collision-free Communication Mechanism

In this section, we discuss a collision-free communication mechanism for radio networks with grid topology. It is based on results concerning 2-distance coloring of grids that have been proposed by Fertin et al. in [5]. A 2-distance coloring of a graph is a proper coloring of vertices satisfying that no vertices in distance at most 2 have assigned the same color. Hence no vertex has in its neighborhood two vertices with the same color. Applying algorithm for computing grid coordinates, we may assume that each node is aware of its grid coordinates.

Definition 4. *Let $[x, y]$ be the coordinates of a node v. The number $TR(v) = (2x + y) \ mod\, 5$ is called a collision free number of the node v.*

Collision free mechanism is defined as follows. A node v is allowed to transmit a message in the i-th round iff $i \equiv TR(v)\ (mod\, 5)$. Correctness of this mechanism follows from that fact that $TR(v)$ corresponds to a 2-distance coloring of a grid with 5 colors, which is moreover shown to be optimal in [5]. Thus we can use algorithms that are not primary designed for the communication in radio networks. The slowdown caused by the presented mechanism is by a constant factor.

5 Conclusion

We discussed communication in radio networks in the case when underlying reachability graph is a grid graph. We presented asymptotically optimal broadcasting algorithm. Moreover, the energetic complexity is asymptotically optimal, too. The algorithm has been modified and we obtained two another algorithms. The first one realizes the task of acknowledged broadcasting. The second one solves the problem of computation of grid coordinates in an unknown grid network. The later algorithm can be applied to compute an initial information for a collision-free communication mechanism. Note that the algorithm for computation of grid coordinates of nodes can be extended to compute dimensions of the underlying grid graph. Another extension could be an algorithm that solves the task of assignment of compact identifiers to the nodes (i.e. the nodes have to be labelled by unique numbers from the set $1, \ldots, n$).

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