Generating and cataloging symmetric graphs*

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Abstract

Symmetry is a concept which plays a significant role in many areas of human activity. In mathematics, the desire to understand symmetry gave birth to modern group theory. Even today, groups are often studied in terms of their representations as symmetry groups of fixed mathematical objects, such as graphs. The study of groups acting on graphs, especially highly symmetric graphs, have resulted in many deep theories and ground-breaking results throughout mathematics. When studying a class of discrete objects, it is of profound importance to be able to construct complete lists of the representatives of this class up to a given size. On one hand, such lists (or censuses, as are often called) help us formulate and test conjectures. On the other hand, the lack of practical means of constructing such censuses indicates the lack of understanding of the theory. Thus, development of theory (and, of course, computational resources) enables constructions of complete censuses of objects, while censuses themselves facilitate and motivate further theoretical achievements. Attempts of constructing census of graphs with high level of symmetry began in early 1930s, when Foster started collecting examples of arc- transitive graphs of valence 3. His work, now known as Foster's census, has been a valuable source of information for graph and group theorists for many decades. Several legendary mathematicians have been involved in constructions of catalogues of graphs of specific symmetry types, such as William Tutte, Harold Coxeter, John Conway etc. In the last few decades, Foster's original work was successfully upgraded in several directions. The aim of the talk is to present the theory and methods behind constructions of these classical censuses, give a few practical demonstrations using modern computer algebra systems, and present some of the more recent achievements in this area.

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